Bayesian inference with an adaptive proposal density for GARCH models

T. Takaishi[a]

[a]Hiroshima University of Economics
5-37-1 Gion, Asaminami-ku, Hiroshima 731-0192, Japan
takaishi@hiroshima-u.ac.jp

We propose a new algorithm to construct a proposal density used in the Bayesian inference of GARCH models. The proposal density is constructed adaptively by using data pre-sampled by a Markov Chain Monte Carlo (MCMC) simulation. We find that our method significantly reduces correlations among the sampled data and its efficiency is comparable with the state-of-the-art AR/MH algorithm[1,2].

In financial time series modeling, a popular model used in literature is the GARCH model[3,4] which can mimic some of the volatility properties seen in real financial markets. A conventional approach to infer GARCH model parameters is the maximum likelihood (ML) estimation. An alternative approach, which recently becomes popular, is the Bayesian inference. The Bayesian inference is usually performed by MCMC methods. In recent studies[3,4] it is shown that Acceptance-Rejection/Metropolis-Hastings (AR/MH) algorithm works better than other MCMC algorithms. In the AR/MH algorithm the proposal density is assumed to be a multivariate Student's t-distribution (STD) and its parameters are estimated by the ML technique. Here we develop a method to determine the parameters without relying on the ML method. We construct the proposal density adaptively by updating its parameters during a MCMC simulation. We call our method "adaptive construction method".
The GARCH(1,1) model to the time series data y_t is given by

$$y_t = \sigma_t \epsilon_t, \tag{1}$$

$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{2}$$

where α, β and ω are the GARCH parameters to be inferred, and ϵ_t is taken from the normal distribution $N(0,1)$.
To test the adaptive construction method we use artificial GARCH data generated with a known parameter set, $\alpha = 0.1$, $\beta = 0.8$ and $\omega = 0.1$. For this artificial data we perform MCMC simulations. We use a (3-dimensional) multivariate STD given by

$$g(\theta) = \frac{\Gamma((\nu+3)/2)/\Gamma(\nu/2)}{\det \Sigma^{1/2}(\nu\pi)^{3/2}} \left[1 + \frac{(\theta - M)^t \Sigma^{-1}(\theta - M)}{\nu} \right]^{(\nu+3)/2)}, \tag{3}$$

where $\theta = (\alpha, \beta, \omega)$ and M are column vectors, and $M_i = E(\theta_i)$. Σ is a covariance matrix and ν is a parameter to tune the shape of STD. This $g(\theta)$ is used as a proposal density in the MH algorithm.
Implementation of the adaptive construction method is as follows. First we start a run by the Metropolis algorithm. The first 3000 data are discarded as burn-in process. Then we accumulate 1000 data for M and Σ estimations. The estimated M and Σ are substituted to $g(\theta)$ of eq.(3). After that we re-start a run by the MH algorithm with the proposal density $g(\theta)$. Every 1000 update we re-calculate M and Σ and update $g(\theta)$. By doing that, we can reach the accurate M and Σ values. To check ν parameter dependence on the MCMC estimations we use $\nu = (4, 6, 8, 10, 12, 20)$ and perform the same MCMC simulation for each ν. We find that ν dependence on the MCMC results is weak. Therefore the results from $\nu = 10$ simulations will be mainly shown.

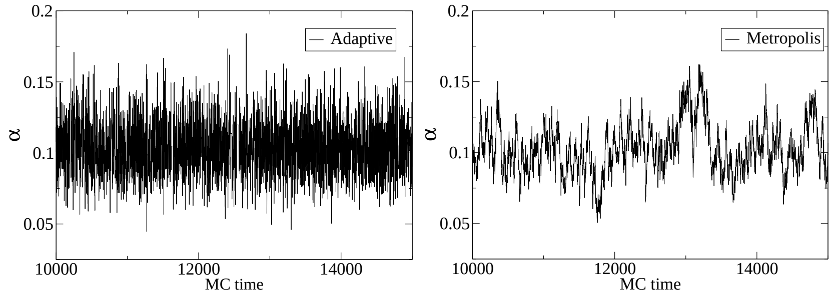

Figure 1: Monte Carlo histories of α from the adaptive construction method with $\nu = 10$(left) and the Metropolis algorithm(right).

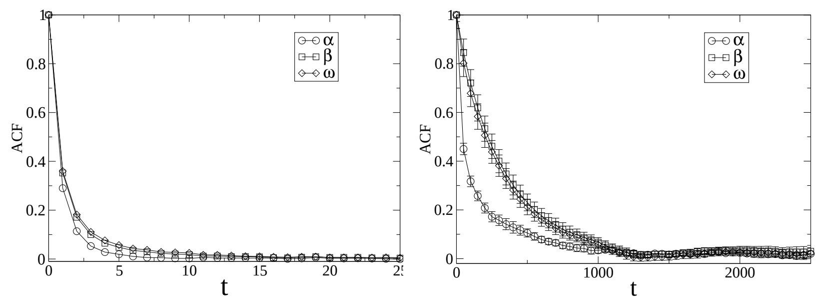

Figure 2: ACF for the adaptive construction method with $\nu = 10$ (left) and the Metropolis algorithm (right).

Fig. 1 compares the Monte Carlo history of α generated by the adaptive construction method with that by the Metropolis algorithm. It is clearly seen that the data α sampled by the adaptive construction method are well de-correlated. For β and ω we also see the similar behavior.

Fig. 2 shows the autocorrelation functions (ACF) for the adaptive construction method and the Metropolis algorithm. The ACF of the adaptive construction method decreases quickly as Monte Carlo time t increases. The autocorrelation time (ACT) of the adaptive construction method is estimated to be $2-3$, which is considerably small. This ACT is similar to that of the state-of-the-art AR/MH method[2]. Thus it is claimed that the efficiency of the adaptive construction method is comparable to that of the AR/MH method. We conclude that the adaptive construction method serves as an alternative efficient MCMC method for GARCH parameter inference.

Keywords

Bayesian inference, Markov Chain Monte Carlo, GARCH model, time series analysis

References

[1]H. Mitsui and T. Watanabe, "Bayesian analysis of GARCH option pricing models," *J. Japan Statist. Soc. (Japanese Issue) 33*, 307-324, 2003

[2]M. Asai, "Comparison of MCMC Methods for Estimating GARCH Models," *J. Japan Statist. Soc. 36*, 199-212, 2006

[3]R.F. Engle, "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance," *Econometrica 60*, 987-1007, 1982

[4]T. Bollerslev, "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics 31*, 307-327, 1986