Non-Gibrat's law and growth rate distributions

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There are many studies for the growth rate distribution of firms and Pareto's law [1]. Recently, Fujiwara et al. [2] show that Pareto's law is derived from the law of detailed balance and Gibrat's law [3]. The detailed balance is time-reversal symmetry ($x_1 \leftrightarrow x_2$) observed in a stable economy

$$P_{12}(x_1, x_2) = P_{12}(x_2, x_1). \tag{1}$$

Here, x denotes a company size such as wealth, sales, profits, the number of employees and so forth, and x_1, x_2 are two successive those, and $P_{12}(x_1, x_2)$ is the joint probability density function (pdf). Also, we define a growth rate as $R = x_2/x_1$. Gibrat's law means that the conditional pdf $Q(R|x_1)$ of the growth rate R is independent of the past value x_1 in large scale region

$$Q(R|x_1) = Q(R) \qquad \text{for} \qquad x > x_0, \tag{2}$$

where x_0 is a certain threshold.

On the other hand, $Q(R|x_1)$ depends on the past value x_1 in the middle scale region (for instance [4]). In Ref. [5], it is reported that the detailed balance and the assumption of the exponential tent-shaped growth rate distribution

$$Q(R|x_1) = \text{Const.} R^{-1\mp t_\pm(x_1)} \qquad \text{for} \qquad R \gtrless 1 \tag{3}$$

lead a Non-Gibrat's law

$$t_\pm(x_1) = t_\pm(x_0) \pm \alpha \ln \frac{x_1}{x_0}, \tag{4}$$

where $\alpha = 0$ for $x > x_0$ and $\alpha \neq 0$ for $x < x_0$. According to Eq. (4), the probability of the positive growth decreases and the probability of the negative growth increases symmetrically as the classification of x increases in the middle scale region (Fig. 1). It is confirmed that these size dependences (3) and (4) are consistent with profits data in Japanese firms [5].

On the contrary, as for sales or assets of firms, the probability of the positive and negative growth rate decrease simultaneously as the classification of x increases [6] as shown in Fig. 2. This is not appropriate to Eq. (4). This is caused by the functional form of Eq. (3). Equation (3) is a linear approximation on logarithmic scales

$$\log_{10} q(r|x_1) = c \mp t_\pm(x_1)\, r \qquad \text{for} \qquad r \gtrless 0, \tag{5}$$

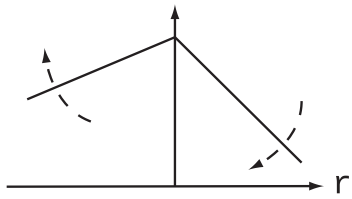

Figure 1: The growth rate pdf for profits of firms on logarithmic scales.

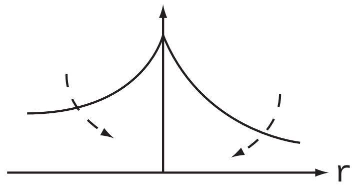

Figure 2: The growth rate pdf for sales or assets of firms on logarithmic scales.

where $q(r|x_1)$ is the conditional pdf of lagarithmic growth rate $r = \log_{10} R$. For sales or assets of firms, these linear approximations are not suitable because those growth rate pdf have curvature [2].

In this study, we take account of the curvature and add a second order terms of r:

$$\log_{10} q(r|x_1) = c \mp t_\pm(x_1)\ r + \ln 10 u_\pm(x_1)\ r^2 \qquad \text{for} \qquad r \gtrless 0. \tag{6}$$

These are rewritten in terms of $Q(R|x_1)$

$$Q(R|x_1) = \text{Const.}\ R^{-1\mp t_\pm(x_1)+u_\pm \ln R} \qquad \text{for} \qquad R \gtrless 1. \tag{7}$$

In this case, the detailed balance is expressed as

$$\frac{P(x_1)}{P(x_2)} = \frac{1}{R}\frac{Q(R^{-1}|x_2)}{Q(R|x_1)} = R^{\ 1+t_+(x_1)-t_-(x_2)-[u_+(x_1)-u_-(x_2)]\ln R} \tag{8}$$

for $R > 1$. After differentiating Eq. (8) with respect to R and setting $R = 1$, differential equations for $t_\pm(x_1)$ and $u_\pm(x_1)$ are obtained. By solving them $t_\pm(x_1)$ and $u_\pm(x_1)$ are determined with several parameters. We find parameter sets consistent with the feature of the growth rate distribution for sales or assets of firms (Fig. 2). In the presentation, we show the detail of the above solutions, and discuss cases in which $\log_{10} q(r|x_1)$ includes higher-order of r.

Keywords
Growth rate, (Non-)Gibrat's low, detailed balance, Pareto's low

References
[1] V. Pareto, *Cours d'Economique Politique* (Macmillan, London, 1897).
[2] Y. Fujiwara, W. Souma, H. Aoyama, T. Kaizoji and M. Aoki, Physica **A321** (2003), 59. Y. Fujiwara, C.D. Guilmi, H. Aoyama, M. Gallegati and W. Souma, Physica **A335** (2004), 197.
[3] R. Gibrat, *Les inegalites economiques* (Sirey, Paris, 1932).
[4] M.H.R. Stanley, L.A.N. Amaral, S.V. Buldyrev, S. Havlin, H. Leschhorn, P. Maass, M.A. Salinger, H.E. Stanley, Nature **379** (1996), 804.
[5] A. Ishikawa, Physica **A367** (2006), 425; Physica **A383/1** (2007), 79.
[6] T. Mizuno, M. Takayasu and H. Takayasu, Physica **A332** (2004), 403.