Multiplicative stochastic process satisfying the law of detailed balance and Pareto's law under Gibrat's law

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Some empirical laws are observed in the distribution of wealth, assets, sales, profits, income, the number of employees and so forth (x). Three representative laws are confirmed; Pareto's law [1], Gibrat's law [2] and the law of detailed balance [3]. The Pareto's law means the cumulative probability distribution function $P(>x)$ in the large scale region follows the power-law distribution

$$P(>x) \propto x^{-\mu} \qquad \text{for} \quad x > x_0,$$

where x_0 is a certain threshold. The Gibrat's law means the statistical independence of growth rate R in the large scale region, here $R = x_2/x_1$, $(x_1,\, x_2)$ are two successive x. It is expressed by the conditional probability distribution function $Q(R|x_1)$:

$$Q(R|x_1) = Q(R) \qquad \text{for} \quad x_1 > x_0.$$

The detailed balance is a time-reversal symmetry of joint pdf $P_{12}(x_1, x_2)$ observed in a stable economy $P_{12}(x_1, x_2) = P_{12}(x_2, x_1)$.

In Ref. [4], we have shown that the detailed balance law and the Non-Gibrat's law

$$Q(R|x_1) = \text{Const.}\ R^{\mp t_\pm(x_1)-1} \qquad \text{for} \quad R \gtrless 1$$
$$t_\pm(x_1) = t_\pm(x_0) \pm \alpha \ln(x_1/x_0)$$

lead the log-normal distribution in the middle scale region as follows:

$$P(x) = Cx^{-(\mu+1)} \exp\{-\alpha \ln^2(x/x_0)\} \qquad \text{for} \quad x_{\min} < x < x_0.$$

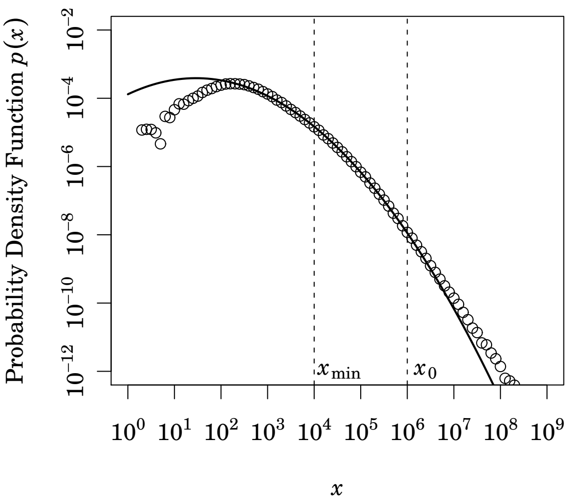
If we build a model based on the detailed balance and (Non-)Gibrat's law, various economic situations can be simulated. This might make the reason clear why the parameters take the empirical values. Furthermore, we can study assets or sales data of firms which are difficult to obtain. In this study, we will propose the simulation model based on the observation of economic data.

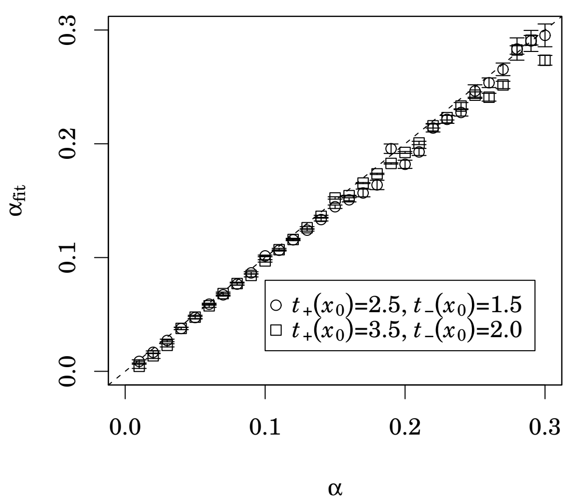
There is a model which leads the Pareto's law by using multiplicative stochastic process. It is called Takayasu-Sato-Takayasu(TST) model [5]. This model is given by the Langevin equation with a multiplicative stochastic noise $b(t)$ and an additive stochastic noise $f(t)$:

$$x(t+1) = b(t)x(t) + f(t).$$

In this study, we have clarified the Gibrat's law is satisfied in the region where the additive noise is negligible. Moreover, we have imposed a condition for the multiplicative noise to satisfy the detailed balance. As a result of the numerical simulation, the detailed balance has been confirmed in a region where influence of the additive noise is ignored.

Furthermore, we have expanded the TST model to satisfy the Non-Gibrat's law. The log-normal distribution in the middle scale region have been confirmed by the numerical simulation (left figure). The parameters α have muched α_{fit} well (right figure), here α are input parameter for Non-Gibrat's low, and α_{fit} are obtained in the simulation result by fitting the log-normal distribution.





Keywords
multiplicative stochastic process, Pareto's low, (Non-)Gibrat's low, detailed balance

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