HIGHER-ORDER PHASE TRANSITIONS ON THE FINANCIAL MARKET

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We thoroughly study the thermodynamic properties of the *anomalous* multifractal structure of random interevent (or intertransaction) times for futures by using (extended in our recent work [1]) Continuous-Time Random Walk (CTRW) formalism of Montroll-Weiss [2-4], as well as Scher and Lax [5,6]. Although the approach is quite general (and can be applied to any interhuman communication having nontrivial priority) we consider it in the context of the financial market where heterogeneous agent activities can occur within a wide spectrum of time scales. We found as the main, general consequence that within this extended formalism the scaling power-dependent partition function, $Z(q)$, diverges for any negative scaling powers q (which justify the name *anomalous*), while for the positive ones it possesses scaling with exponent $\tau(q)$ which is a nonanalytic (singular) function of q. In the definition of the partition function we used the pausing-time distribution as the central one, which has the form of a convolution (or superstatistics used, e.g., for the description of turbulence [7,8] as well as a speculative market [9]). Its integral kernel is given by the stretched exponential distribution (often used in disorderd systems). This is an intermediate one between the exponential distribution assumed in the original version of the CTRW formalism (for description of the transient photocurrent measured in amorphous glossy material [3,6]) and the Gaussian one sometimes used in this context (e.g. for diffusion of hydrogen in amorphous metals and for aging effects in glasses). A more rafined but heuristic analytical prediction was also considered. We argue that this superstatistics defines a kind of non-geometric random multiplicative cascadic process (while the geometric one was used, e.g., in the fully developed turbulence) which says how the investor activities are spreading among different scales ruled by fluctuations. As the most important result we found (by using the saddle-point approximation) the third- and higher-order phase transitions which can be roughly interpreted as transitions between the phase in which high frequency trading is most visible and the phase defined by the low frequency trading; the order of the phase transition depends directly on exponent α defining the stretched exponential distribution.

Keywords
Continuous-Time Random Walk, multifractality, thermodynamics, power-dependent partition function, Forex market, futures,

References

[1] J. Parelló, J. Masolivar, A. Kasprzak, and R. Kutner, Phys. Rev. E 78 (2008) 036108-036119 .
[2] E. W. Montroll and G. H.Weiss, J. Math. Phys. 6 (1965) 167-181.
[3]E. W. Montroll and M. F. Shlesinger, "On the wonderful world of random walks", in *Nonequilibrium Phenomena II. From Stochastics to Hydrodynamics*, SSM Vol. XI, p. 1-121, 1984.

[4] G. H. Weiss, “A Primer of Random Walkology”, in *Fractals in Science*, (Springer-Verlag, Berlin), p. 107-161.
[5] H. Scher and M. Lax, Phys. Rev. B7 (1973) 4491-4519.
[6] G. Phister and H. Scher, Adv. Phys. 27 (1978) 747-798.
[7] B. Castaing, Y. Gagne, E. Hopfinger, Physica D 46 (1990 177-200.
[8] B. Chabaud, A. Naert, J. Peinke, F. Chillà, B. Castaing, B. Hebral, Phys. Rev. Lett. 73 (1994) 3227-3230.
[9] J. F. Muzy, J. Delour, E. Bacry, Eur. Phys. J. B 17 (2000) 537-548.