The difference of growth rate distributions between sales and profits

A. Ishikawa[a] and **S. Fujimoto**[b]

Faculty of Business Administration and Information Science, Kanazawa Gakuin University
10 Sue-machi, Kanazawa, Ishikawa 920-1392, Japan
[a]ishikawa@kanazawa-gu.ac.jp, [b]fujimoto@kanazawa-gu.ac.jp

In the large scale region of assets, sales, profits, income of firms, personal income and so forth (denoted by x), Pareto's law is often observed [1]. In recent years, it is clarified that the Pareto's law can be derived from the law of detailed balance and Gibrat's law, which are also observed in the data [2]. Here, the detailed balance is time-reversal symmetry of the joint probability density function (pdf) of two values at two consecutive years (denoted by x_1 and x_2). Gibrat's law means that the conditional pdf of the growth rate $R = x_2/x_1$, on the condition that the firm size x_1 in an initial year is fixed, is independent of x_1 [3]. Following this study, it is also found that the log-normal distribution can be deduced from the detailed balance and Non-Gibrat's law in the middle scale region [4]. Here, Non-Gibrat's law means a statistical dependence of the growth rate distribution on the past value x_1 in the middle scale region.

These findings are avail in themselves. Furthermore, it is also interesting that the form of the growth rate distribution of assets, sales of firms or personal income (Fig. 1) is different from that of profits or income of firms (Fig. 2). This difference is observed in the analysis of (Non-)Gibrat's law. In Figs. 1 and 2, the horizontal axis is the logarithm of growth rate $r = \log_{10} R$ and the vertical axis is the logarithm of its pdf. Figure 2 represents that the growth rate distribution is approximated by linear functions in both positive and negative r in log-log scale. On the other hand, Fig. 1 means that the growth rate distribution is wider than the linear approximation.

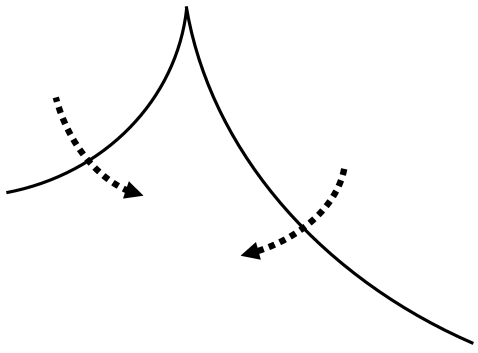

Figure 1: The growth rate distribution of assets, sales of firms or personal income. The horizontal axis is the logarithm of growth rate $r = \log_{10} R$ and the vertical axis is the logarithm of its pdf.

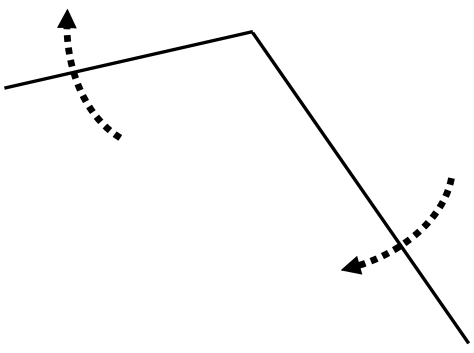

Figure 2: The growth rate distribution of profits or income of firms. The horizontal axis is the logarithm of growth rate $r = \log_{10} R$ and the vertical axis is the logarithm of its pdf.

This difference probably arises from the fact that assets, sales of firms and personal income are positive quantities, but profits and income of firms are both positive and negative quantities. In this study, we analyze sales and profits data of Japanese firms in 2003 – 2007 years [5] in order to confirm this assumption. The growth rate distributions of sales and profits

x themselves are depicted in Fig. 1 and 2, respectively. We confirm that the growth rate ($S = v_{23}/v_{12}$) distributions of the difference of sales and profits ($v_{12} = x_2 - x_1$, $v_{23} = x_3 - x_2$) are depicted in Fig. 2 in both cases. This observation is one of exhibits with the assumption. Because sales are positive quantities themselves, but the difference of sales can be not only positive but also negative.

From the growth rate distribution of the difference $v_{12} = x_2 - x_1$ (Fig. 2) observed in empirical data, it is conceivable to derive followings analytically or numerically. (a) The growth rate distribution of x which cannot be negative (assets, sales of firms and personal income) is depicted in Fig. 1. (b) The growth rate of distribution x which can be negative (profits and income of firms) is depicted in Fig. 2. In addition, the difference of Non-Gibrat's laws between Fig. 1 and 2 might be clear. In Fig. 2, the probability of the positive growth decreases and the probability of the negative growth increases symmetrically as the classification of x increases in the middle scale region. On the other hand in Fig. 1, the probability of the positive and negative growth decrease simultaneously as the classification of x increases.

Keywords
growth rate, firms, sales, profits

References

[1] V. Pareto, *Cours d'Economique Politique* (Macmillan, London, 1897).
[2] Y. Fujiwara, W. Souma, H. Aoyama, T. Kaizoji and M. Aoki, Physica **A321** (2003), 59.
Y. Fujiwara, C.D. Guilmi, H. Aoyama, M. Gallegati and W. Souma, Physica **A335** (2004), 197.
[3] R. Gibrat, *Les inegalites economiques* (Sirey, Paris, 1932).
[4] A. Ishikawa, Physica **A383/1** (2007), 79.
A. Ishikawa, Physica **A371** (2006), 525.
[5] TOKYO SHOKO RESEARCH, LTD., http://www.tsr-net.co.jp/.