Random matrix theory filters and currency portfolio optimisation

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Random matrix theory (RMT) filters, applied to covariance matrices of financial returns, have recently been shown to offer improvements to the optimisation of financial portfolios. This paper studies the effect of three RMT filters on realised portfolio risk, using *bootstrap* analysis and *out-of-sample* testing, in the case of a typical foreign exchange and commodity portfolio, weighted towards foreign exchange, and consisting of $N = 39$ assets. This is intended to test the limits of RMT filtering, which is more obviously applicable to portfolios with large numbers of assets. The filters examined were those of Laloux et al. [1], Plerou et al. [2], and Daly et al. [3], referred to here as the LCPB, PG+ and KR filters respectively. Each filter was applied to both equally and exponentially weighted correlation matrices. This analysis was performed using 8 years and 10 months of recorded daily market activity.

For the in-sample analysis, and following [4], bootstrapped samples were taken, and the mean across all samples, of the realised risk of the forecast minimum risk portfolio, was assessed, as shown in Figure 1. As in the S&P case [3], this showed, in general, the potential of RMT filters to reduce realised risk. Notably, in the equally weighted case, a preference was shown for the lowest available number of past moves, T, to be used (equivalent to the lowest Q value), in conjunction with RMT filtering. In the exponentially weighted case, RMT filtering was again preferred overall, and the optimal in-sample decay factor, α, coincided with the Riskmetrics [5] recommendation of 0.97, in contrast to the S&P case [3].

For comparing these models out-of-sample we used forward validation, where the value of the weighting parameter (α or T), and the choice of stability-based (KR) filter, were determined out-of-sample. Table 1 summarises the out-of-sample performance, with RMT filtering seen *on average* to reduce realised risk in all cases, compared to the unfiltered portfolio. We further observed that RMT filtering reduced mean realised risk in the majority of individual years. However, while realised risk was reduced on the majority (62.6%) of individual days, the filters were also found capable of increasing realised risk substantially on any one day. These results are consistent with recent results for a much larger S&P 500 portfolio [3]. Out-of-sample, the exponential weightings again showed good consistency with the value of 0.97 suggested by Riskmetrics [5], in contrast to previous results involving stocks [3,4]. These decay factors, together with the low number of past moves preferred in the filtered, equally weighted case, displayed a trend, over the time period tested, towards models which were more reactive to recent market changes.

When RMT filtering was applied to foreign exchange and commodity portfolios with *fewer* asset numbers it was observed, in general, that the benefit of filtering was reduced as asset numbers (N) decreased. In some cases filtering provided *no overall risk reduction*. This was also reflected in the out-of-sample filter performance, for a portfolio consisting of 15 major currencies and commodities. In this case, RMT filtering provided no long term risk reduction, and was more likely to *increase* realised risk, both overall and on any individual day.

Taken as a whole, our results suggest that RMT filtering can provide risk reduction for foreign exchange portfolios involving *sufficient numbers of assets*. Moreover, RMT filters uncovered different uses of models than were possible with unfiltered analysis, namely ones that reacted quickly to market events. Without filtering these features, which utilise very recent data, were found to be masked by noise.

Table 1: Mean out-of-sample realised risk as a percentage of that for unfiltered equally weighted covariance, for the full foreign exchange and commodity portfolio, with 39 assets.

Model	Unfiltered	LCPB	PG+	KR
Equal Weights	100	87	85.8	86.6
Exponential Weights	98.1	91.8	92.7	87.4

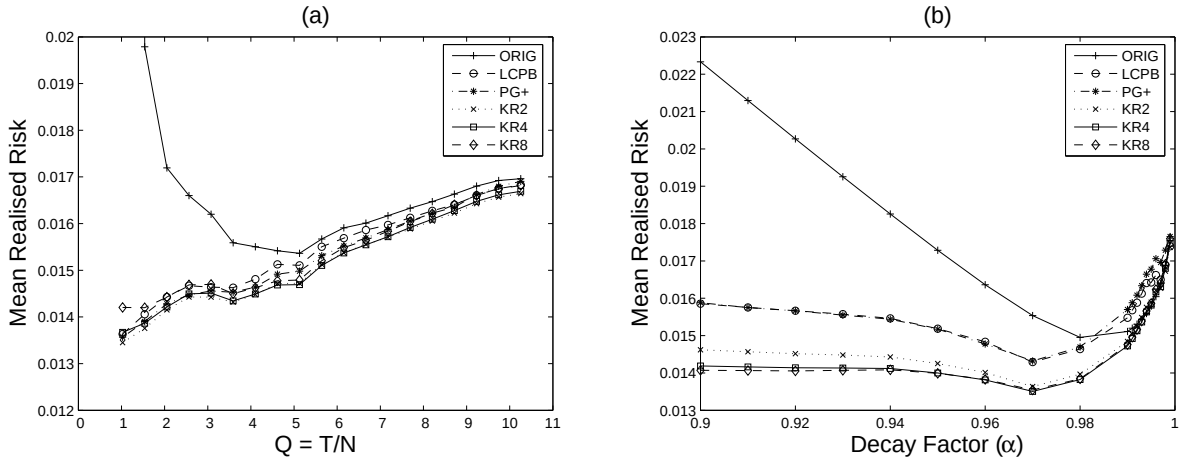

Figure 1: Mean bootstrapped (in-sample) realised risk, for selected filters, applied to (a) equally weighted and (b) exponentially weighted volatility forecasts, and for unfiltered volatility ("ORIG"), for the full foreign exchange and commodity portfolio, with 39 assets.

Keywords
foreign exchange, portfolio optimisation, random matrix theory, risk analysis

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