

Zipf's Law for All Cities

H. Rozenfeld^a, D. Rybski^a, X. Gabaix^b, H. Makse^a

^aLevich Institute and Physics Department, City University of New York,
New York, NY 10031. USA.

^bStern School, New York University
New York, NY 10012. USA

Zipf's law for cities is a remarkable regularity in economics, stating that the probability that a city has a population S is a power law with exponent 2. Zipf's law has been documented for many countries, but always for a very limited number of cities; no more than a few hundreds cities. The reason for this is that it is hard and time-consuming to define meaningful cities, that put together economically related entities (such as Boston and Cambridge, MA). Hence, whether Zipf's law holds outside the very upper tail of the city distribution is so far unknown. Furthermore it has been conjectured that it fails, as administratively defined (rather than geographically defined) cities follows a log-normal distribution rather than a power law. Here, we use a recently proposed clustering algorithm, the CCA to construct cities from the bottom-up. We construct cities without using administrative data, and use instead geographical proximity to construct cities. We find that Zipf's law holds for cities above 10,000 inhabitants in the US, and above 1,000 inhabitants in the UK. Our results bring an answer to the nearly one-century old question of the distribution of city sizes and present severe constraints on theories of cities.

Keywords

Cities, population, population distribution, Zipf's Law, clustering algorithm.