Scaling of the volatility of growth rates in macroeconomics and finance

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In economics, as well in the natural sciences, the study of volatility scaling plays an important role in describing the relationships between "micro" and "macro" levels. In economics, the exact functional dependence between the volatility of growth rates of GDP and country size was not know until Canning et al [1] found that the standard deviation of the logarithmic growth rate R decreases with increasing GDP as a power law $\sigma(R) \propto GDP^{\beta}$, with a scaling exponent $\beta \approx -0.15$. We speculate that the scaling mechanism found in the volatility of GDP growth rate also exists in the scaling of growth rate of different macroeconomic variables $S_{a,I,t}$, where $a = 1$ stands for public debt, $a = 2$ for government consumption expenditure, 3 for public health expenditure, 4 for exports of goods, 5 for total imports of goods and services, and $a = 6$ for total exports of goods and services, for different countries I and different years t. For each $S_{a,I,t}$, we calculate the annual growth rate of country I at year t as $R_{a,I,t} = \ln(S_{a,I,t+1}/S_{a,I,t})$. In order to determine the scaling law for the volatility of growth rate residuals r_a, we define

$$R_{a,I,t} \equiv \mu_{a,I} + r_{a,I,t}, \tag{1}$$

where $\mu_{a,I}$ is the expected growth rate of S_a in country I. In order to improve statistics, as the number of data points for each country is very limited, we join all $(r_{a,I,t}, S_{a,I,t})$ pairs for each variable S_a into one common data set.

We first qualitatively investigate for each economic variable S_a how the growth rate probability distribution function (pdf) depends on the size of S_a. For each S_a we sort the data set into three subsets of equal size, low S_a, medium S_a, and high S_a. For each subset S_a, we find that the pdf of residuals $P(r_a)$ is exponential rather than normal. In Fig. 1 we plot the empirical pdf $P(r_a)$ for the smallest and largest groups, for public debt ($a = 1$) and government consumption expenditure ($a = 2$). The pdfs are plotted on a log scale to emphasize that the absolute value of the residuals are double exponential. If the pdf $P(r_a)$ were normally distributed the pdf would be a quadratic function of r_a on a log scale. Next we find qualitatively that for each variable S_a studied, the growth rate residuals r_a are heteroscedastic since the standard deviation $\sigma(r_a)$ changes (decays) with S_a. In Fig. 1 we show for $a = 1$ and $a = 2$ that the residuals r_a obtained for countries with high S_a values have a smaller $\sigma(r_a)$ than the residuals calculated for countries with small S_a values.

In order to test quantitatively how the volatility of growth rate changes with the size of S_a, for each S_a we subdivide the whole sample into ten equal subintervals of log S_a. Then we find that, for each economic variable S, the standard deviation $\sigma(r_a)$ of the growth rate residuals r_a versus the size of S_a in the corresponding interval. For each variable we find

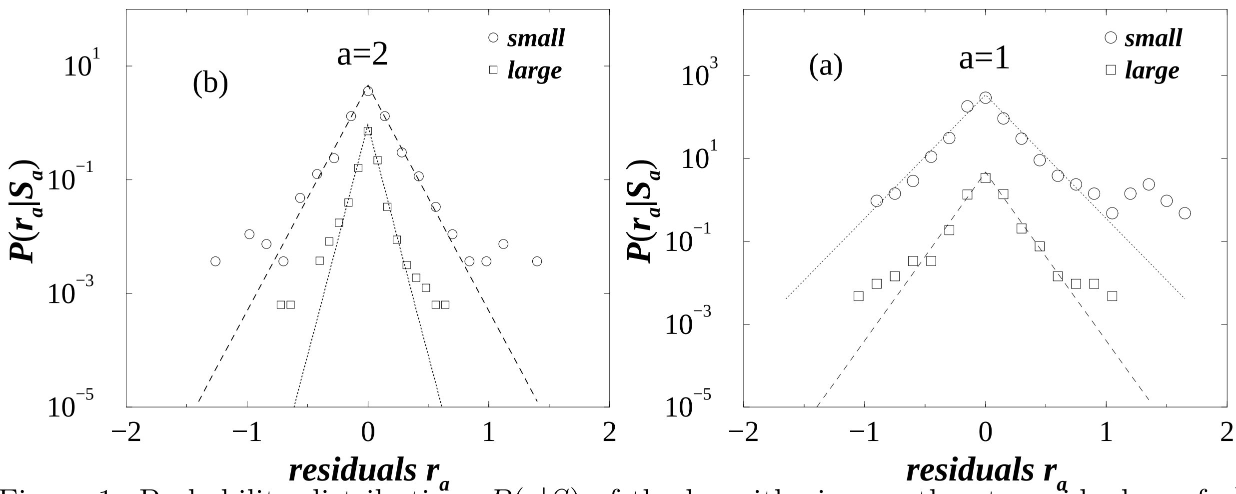

Figure 1: Probability distributions $P(r|S)$ of the logarithmic growth rate residuals r of a) public debt and b) government consumption expenditure for two different ranges of S.

that the standard deviation $\sigma(r_a)$ scales with its size as a power law with a scaling exponent surprisingly close to that found for GDP.

Next we accomplish similar analysis in finance. It is known that both risk and return (growth rate) generally decrease with increasing firm size [3], but what are the functional dependences? Does risk or return decay faster? To address these questions quantitatively, we analyze—for a large set of stocks comprising the New York Stock Exchange Composite—the annual logarithmic growth rate R and the firm size, quantified by the market capitalization MC. We find that the pdf $P(R)$ follows a Laplace distribution in the broad central region with a standard deviation $\sigma(R)$ that decreases with MC as a power law, consistent with stochastic properties found in microeconomics [2] and macroeconomics [1]. For the NYSE Composite we find that the power laws relating $\sigma(R)$ and MC exhibit stability in both the functional form and the exponent. For the S&P500 index (part of the NYSE Composite), we show that the average growth rate decreases faster than standard deviation with MC, implying that the average return-to-risk ratio decreases with MC. These results are potentially valuable. Indeed, in the approach where both average growth rate and risk are considered, a famous statement "All the eggs should not be placed into the same basket" can now be extended according to our results: "Better choose the smaller baskets."

References

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