Analysis and simulation of market dynamics with an extended Minority Games

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The modeling of financial markets and agent-based simulations have attracted a significant amount of attention in recent years.[1] One of the proposed market models is a multi-agent repeated game called Minority Game (MG)[2]. Grand Canonical Minority Game (GCMG) is an extension of MG and earlier studies have succeeded in reproducing the Stylized Facts of financial markets as critical phenomena of the model[3]. To reach the critical region of GCMG, one has to carefully choose a set of model parameters. In other words, GCMG dose not include a self-induced mechanism for the emergence of the stylized facts. In this research, we will first analyze the mechanism behind the GCMG's recovery of stylized facts. An extended MG model with the self-induced approach to the critical state will be further developed based on this mechanism.

In an MG model, each of the N agents has S strategies. Agents choose 1 or -1 in order to win the game. The recent m-step history of the winning side $\mu\{1,2,\cdots,P\}$ is provided as the global information. Based on this information agents will make their decision on the next action using available strategies. Strategies are evaluated by scoring, and agents will use the best scored strategy. GCMG differs from MG in two aspects. First, there are N_P non-adaptive agents called producers who have only a single strategy. Meanwhile there are N_s adaptive agents called speculators. Second, a speculator has a zero-strategy (if active, the agent will stop trade) which receives ϵ points at every time step. Fig.1 shows the cumulative distribution function and the autocorrelation function of price returns in the GCMG market. Both the fat-tailed distribution and the volatility clustering of price returns are found. Fig.2 shows the phase diagram of GCMG with control parameters $\alpha = \frac{P}{N_s}$ and ϵ. Stylized Facts emerge in the vicinity of ϵ=0. However, ϵ=0 itself can be identified as a singular point where no volatility clustering occurs there.

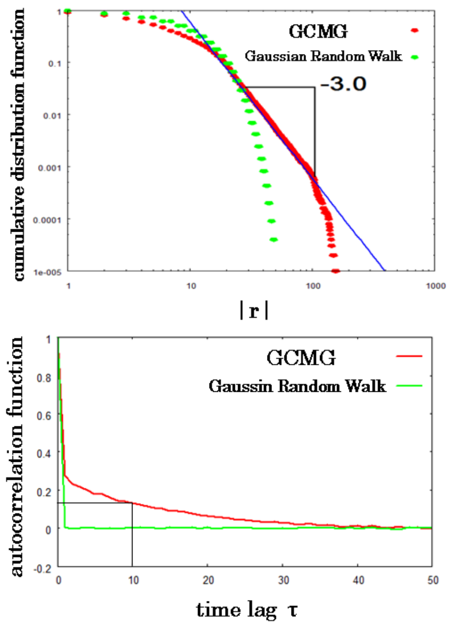

Fig.1 Reproducing the Stylized Facts by GCMG. Fat tails (upper panel) and Volatility clustering (lower panel). Results of GCMG are depicted in red and Gaussian Random Walk in green. (P=10, N_p=20, S=1, α=0.031, ε=0)

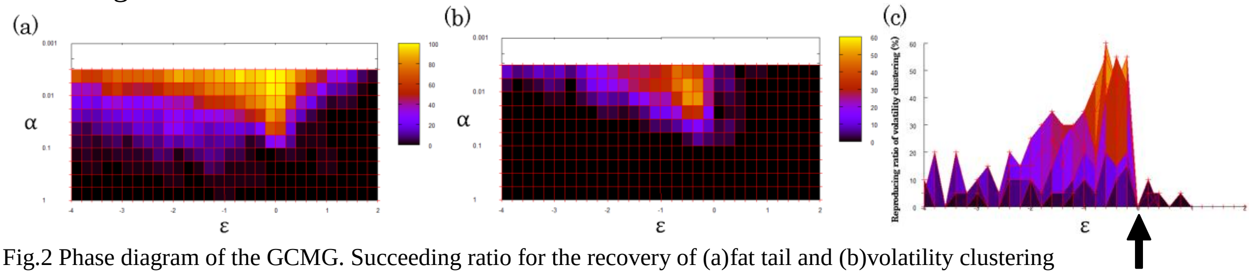

Fig.2 Phase diagram of the GCMG. Succeeding ratio for the recovery of (a)fat tail and (b)volatility clustering are shown. Another view of the phase diagram (c) shows the volatility clustering dose not appear at ϵ=0.

To explain why large price change and long term correlation emerge in the GCMG simulation, we show in Fig.3 the relation between the number of agents who alternate their statuses as active and inactive traders and the frequency of the emergence of volatility clustering. When volatility clustering occurs in the simulation, there seem to be more agents switching their roles.

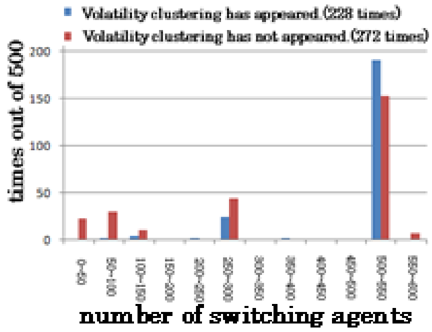

Fig.3 The relation between the number of switching agents and the occurrence of volatility clustering. （P=10, N_p=10, S=1, α=0.01, ϵ=-0.4）

Fig.4 shows the time evolution of scores of active and inactive strategies. Volatility clustering appears only in (c) and (d). This can be explained as follows. It is known that intermittency in fluctuations of the price return is the direct cause for the emergence of the volatility clustering. In case (c), speculators who have low score strategy alternate as active and inactive traders, because the score fluctuations around that of the zero-strategy. When these agents are active, their orders could cancel out the effect of orders issued by agents who have higher scores, since the active strategies are anti-correlated. And when they are inactive, price returns will fluctuate in large amplitudes, since active strategies are positively correlated. Hence the price fluctuates intermittently. In case (d), speculators who have higher score strategies alternate the active and inactive statuses. Strategies of these agents are positively correlated, which could bring the intermittency to the fluctuations of the price returns together with the orders issued by the producers. Case (e) is special in that the anti-correlated strategies can never be active simultaneously. Hence the intermittency in the price return can never occur. It is relatively simple to understand that no intermittency could occur in case (a) and case (b) where either all strategies or no strategy are active.

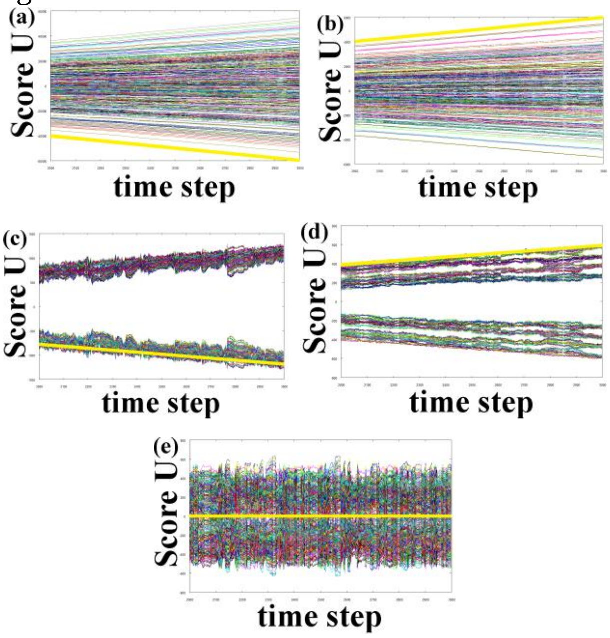

Fig.4 Scores of active strategies (fine lines in a variety of colors) and zero-strategy (thick line in yellow). (a) ϵ=-20.0, (b) ϵ=2.0, (c) ϵ=-0.4, (d) ϵ=0.2, (e) ϵ=0.0. （P=10, N_p=10, S=1, α=0.01）

In the next phase of study, we will propose an extended MG model where ϵ is different among agents and evolves temporally, so that the modeled market autonomously reaches the critical state.

Keywords

Minority Game, Grand Canonical Minority Game, Stylized Facts, intermittency, criticality

References

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