Chain of Bankruptcy on a Nation-wide Production Network

Y. Fujiwara[a], **H. Aoyama**[b], **K. Kobayashi**[c], **Y. Fujita**[a]

[a] NiCT / ATR CIS Applied Network Science Lab
Seika-chou, Kyoto 619-0288, Japan
contact: yfujiwar@atr.jp

[b] Physics Department, Kyoto University
Yoshida, Sakyo-ku, Kyoto 606-8501, Japan

[c] Research Institute of Economy, Trade and Industry
Kasumigaseki, Chiyoda-ku, Tokyo 100-8901, Japan

Production network (PN), or supplier-customer network, is a line of economic activities in which firms buy intermediate goods from upstream firms, put values added on them, and sell the products to downstream firms. A nation-wide PN forms a a giant web of production ranging from upstream to downstream, finally down to consumers, being no less important in real-economy than financing and labor networks.

Supplier-customer link is a credit relation. Whenever one delivers a good to someone else without an immediate exchange of money or goods of full value, credit is extended. Frequently, suppliers provide credit to their customers, who supply credit to their customers and so forth. Once a firm goes into financial insolvency state, its upstream firms are creditors who are not necessarily able to receive the scheduled payment. Then a creditor has its balance-sheet deteriorated in accumulation, and may eventually go into bankruptcy. This is an example of *chain of bankruptcy*.

Total amount of debts of bankrupted firms is not negligible (4% to 6% of nominal GDP in Japan), and a single bankruptcy can have a large debt (distribution of debt when bankrupted follows a Zipf's law). In considering the ripple effect in chain of bankruptcy, therefore, heterogeneity of links and modular structure in PN would play crucial roles. We address this issue and study a real data of the nation-wide network comprising *a million of firms and millions of supplier-customer links* in Japan, and bankruptcies actually occurred on the network [1].

Consider a chain of bankruptcy, defined by a connected component consisting of adjacent bankrupted nodes, and its size. Given a probability p of bankruptcy for a certain period of time, one can calculate the expected size of chained but independent bankruptcy on randomized networks in either the way that nodes of failure are randomly located or in the way that they are fixed at same locations in real data. The former evaluation can be analytically done by using a mean field approximation. Comparison with real data (Fig. 1 (b)) shows that much larger "avalanches" take place than expected by chance. See Fig. 2 for an example.

Because the degree k distribution has a heavy tail (Fig. 1 (a)) , one can expect not only that the number of firms in upstream by 1-link is large, but also that the number by 2-link is larger than one can naively expect. Actually, for a heavy-tailed degree distribution, the latter number n_2 can be estimated by $n_2 = \langle k^2 \rangle - \langle k \rangle$, while naively $n_2 = \langle k \rangle (\langle k \rangle - 1)$, ignoring clustering (transitivity) coefficient ($< 10^{-2}$).

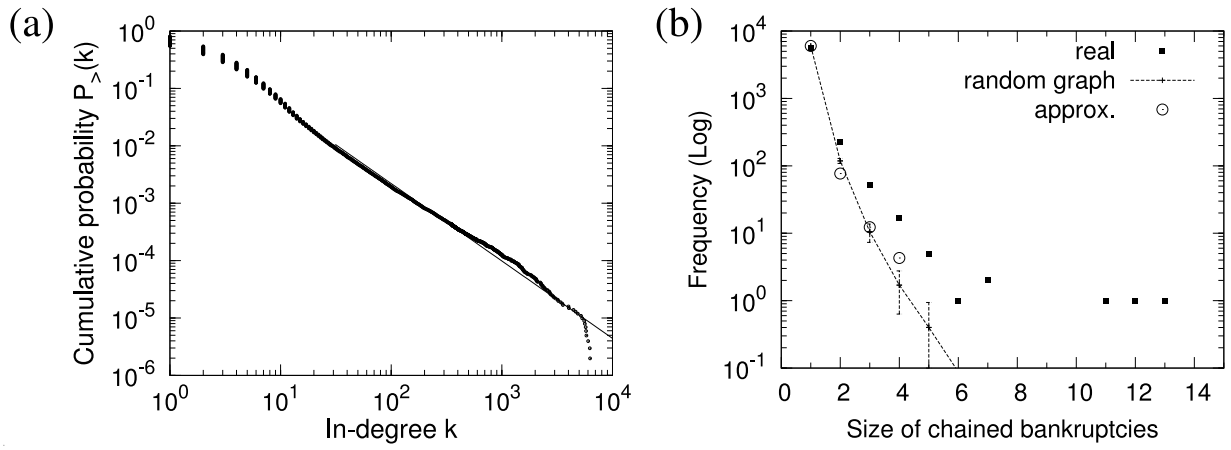

Fig 1. (a) Cumulative distribution $P_>(k_{\rm in})$ of #supplies $k_{\rm in}$; $P_>(k_{\rm in}) \propto k_{\rm in}^{\mu}$ with $\mu \sim 1.35$. (b) Size of chained bankruptcies and frequency (log-scale).

Vulnerable paths would exist for such firms in the upstream side that its out-degree (the number of firms to which they sell their products) is quite small. Since degree and firm-size are usually positively correlated with each other, as checked in out dataset, such vulnerable firms are often small-sized. On the other hand, one may think that large-firms with relatively large numbers of out-degrees might not be influenced much by a bankruptcy in down-stream. However, the fact the number of firms 2-link away is large implies that the vulnerable paths typically influencing small-sized firms are quite possibly abundant in two or more links away. Even if a large-sized firm may be affected little by a single bankruptcy at 1-link away, the ripple effect can affect the firm with a large extent. This suggests a new look at credit-risk management.

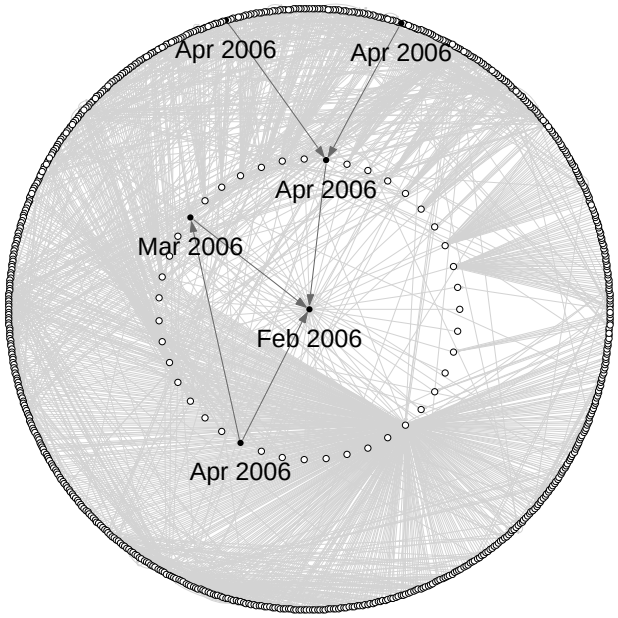

Fig 2. A chain of bankruptcies (black nodes) for size 6. The black nodes belong to construction and related sectors. The dates of bankruptcies are shown, while the nodes in the inner circle are suppliers of the bankrupted firm at the center and those in the outer circle are suppliers of the suppliers.

Keywords
supplier-customer network, chain of bankruptcy, complex network

References

[1] Y. Fujiwara and H. Aoyama, *Large-scale Structure of a Nation-wide Production Network*, http://arxiv.org/abs/0806.4280 .