ECONOMIC FLUCTUATIONS AND STATISTICAL PHYSICS:

QUANTIFYING EXTREMELY RARE AND MUCH LESS RARE EVENTS

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Recent analysis of truly huge quantities of empirical data suggests that classic economic theories not only fail for a few outliers, but that there occur similar outliers of every possible size. In fact, if one analyzes only a small data set (say 10^4 data points), then outliers appear to occur as "rare events." However, when we analyze orders of magnitude more data (10^8 data points!), we find orders of magnitude more outliers—so ignoring them is not a responsible option, and studying their properties becomes a realistic goal. We find that the statistical properties of these "outliers" are identical to the statistical properties of everyday fluctuations. For example, a histogram giving the number of fluctuations of a given magnitude x for fluctuations ranging in magnitude from everyday fluctuations to extremely rare fluctuations that occur with a probability of only 10^{-8} is a perfect straight line in a double-log plot.

Two unifying principles that underlie much of the finance analysis we will present are scale invariance and universality [R. N. Mantegna and H. E. Stanley, *Introduction to Econophysics: Correlations & Complexity in Finance* (Cambridge U. Press, 2000)]. Scale invariance is a property not about algebraic equations but rather about functional equations, which have as their solutions not numbers but rather functional forms—power laws. The key idea of universality is that the identical set of "scaling laws" hold across diverse markets, and over diverse time periods.

An analogy with earthquake research is perhaps appropriate. If one studies limited data sets, an erroneous paradigm arises in which there are everyday (unnoticeable except by sensitive seismometer) "tremors," punctuated from time to time by rare events ("earthquakes"). Thanks to extensive empirical work, we now know that the partition of shocks into "tremors" and "earthquakes" is not valid. Rather, if one examines enough data, one finds that the shocks occur for all possible magnitudes. The law named after Gutenberg and Richter refers to a statistical formula that gives all the data from the smallest tremors to the "big ones"—there are no outliers. Thus, an erroneous paradigm ("normal" size earthquakes occasionally punctuated by "huge" earthquakes) can arise when a limited quantity of data are considered in which data are partitioned into everyday events (often describable by one statistical law) and rare events which, since they are not described by the law, are termed outliers.

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