Recent Developments in the Studies on Financial Volatility

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Volatility, which represents the variance or standard deviation of financial returns, plays a central role in modern finance such as option pricing and Value at Risk. While the well-known Black and Scholes (1973) model assumes that the volatility is constant, few would dispute the fact that the volatility changes over time. Many time series models are now available to describe the dynamics of volatility. One of the most widely used is the ARCH (autoregressive conditional heteroskedasticity) family including ARCH model by Engle (1980), GARCH (generalized ARCH) model by Bollerslev (1986) and their extensions.

Suppose that daily return R_t has the form:

$$R_t = E(R_t|\boldsymbol{I}_{t-1}) + \epsilon_t,\ \ \epsilon_t = \sigma_t z_t,\ \ z_t \sim WN(0,1),$$

where $E(R_t|\boldsymbol{I}_{t-1})$ is the expectation of R_t conditional on the information up to day $t-1$ and z_t is assumed to be white noise with mean 0 and variance 1. Then, σ_t^2 is the variance of R_t conditional on the information up to day $t-1$.

For example, the GARCH$(1,1)$ model specifies the volatility σ_t^2 as:

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\epsilon_{t-1}^2,\ \ \omega > 0,\ \ \beta, \alpha \geq 0,$$

where ω, β and α are parameters, which are assumed to be non-negative to guarantee that volatility is always positive.

The problem of using these models is that we must specify the model before estimating the volatility and the estimate of volatility depends on the specification of volatility. Hence, realized volatility has recently attracted the attentions of financial econometricians. Realized volatility is independent of the specification of volatility dynamics because it is simply the sum of squared intraday returns.

Suppose that the log-price $p(s)$ follows the simple diffusion process:

$$dp(s) = \mu(s)dt + \sigma(s)dW(s),$$

where s is time, $W(s)$ is a standard Brownian process, and $\mu(s)$ and $\sigma(s)$ are the mean and the standard deviation of $dp(s)$ respectively, which may be time-varying but are assumed to be independent of $dW(s)$.

Then, the true volatility for day t is defined as the integral of $\sigma^2(s)$ over the interval $(t-1, t)$, i.e.,

$$IV_t = \int_{t-1}^{t} \sigma^2(s)ds,$$

which is called integrated volatility. In this article, we define $t-1$ and t as the market closing time on days $t-1$ and t respectively.

The integrated volatility is unobservable, but if we have the intraday return data $\left(r_{t-1+1/n},\ r_{t-1+2/n},\ \ldots,\ r_t\right)$, we can estimate it as the sum of their squares

$$RV_t = \sum_{i=1}^{n} r_{t-1+i/n}^2,$$

which is called realized volatility. RV_t will provide a consistent estimate of IV_t, i.e.,

$$\operatorname*{plim}_{n\to\infty} RV_t = IV_t.$$

Hence, the realized volatility will be an accurate estimator of the true integrated volatility as n is sufficiently large or equivalently the time interval of intraday returns used for calculating realized volatility is sufficiently small. However, it should be noted that the presence of microstructure noise such as bid-ask spread and nonsynchronous trading (see Campbell et al. (1997) Chapter 3) prevents the realized volatility from converging to the integrated volatility.

This presentation surveys recent developments in ARCH models and realized volatility. Their applications to volatility forecasting and option pricing are also explained using Watanabe (2007) and Ubukata and Watanabe (2008).

Keywords
ARCH, microstructure noise, option pricing, realized volatility, volatility forecasting

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