# CORRELATION, HIERARCHIES, AND NETWORKS IN FINANCIAL MARKETS

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We discuss methods to quantitatively investigate the properties of correlation matrices of a financial system. Correlation matrices play an important role in portfolio optimization and in several other quantitative descriptions of asset price dynamics in financial markets. Specifically, we discuss how to define and obtain hierarchical trees, correlation-based trees and networks from a correlation matrix. We describe the application of hierarchical clustering and other procedures based on Random Matrix Theory on the correlation matrix as filtering procedures used to detect the statistically reliable aspects of the correlation matrix. As a result of the clustering procedure, a hierarchical tree of the elements of the system is obtained. The correlation-based clustering procedure allows associating a correlation-based network with the correlation matrix [1]. For example, it is natural to select the minimum spanning tree, i.e. the shortest tree connecting all the elements in a graph, as the correlation based network associated with the single linkage cluster analysis. Different correlation based networks can be associated with a hierarchical tree putting emphasis on different aspects of the sample correlation matrix [2]. Examples of correlation-based networks different from the minimum spanning tree are the planar maximally filtered graph [3] and the average linkage minimum spanning tree [4]. We also present a technique developed to associate a value of reliability to the links of correlation-based graphs by using bootstrap replicas of data. We show that both the amount of correlation and the topology of the correlation-based network play a role in determining the bootstrap value measuring the statistical reliability of each link [4]. As an application of the described techniques to real data we examine sets of equity return of highly capitalized stocks traded at major stock exchanges in the world. The information retained in filtering procedures and its stability with respect to statistical fluctuations is quantified by using the Kullback-Leibler distance [5].

**Keywords**
Financial markets, Correlation-based networks, Hierarchical complex systems, Portfolio optimization

**References**

[1] R.N. Mantegna, Hierarchical Structure in Financial Markets, European Physical Journal B 11, 193-197 (1999).

[2] M. Tumminello, F. Lillo, R.N. Mantegna, Correlation, hierarchies, and networks in financial markets, arXiv:0809.4615

[3] M. Tumminello, T. Aste, T. Di Matteo, R.N. Mantegna, A tool for filtering information in complex systems, Proceedings of the National Academy of Sciences of the United States of America 102, 10421-10426 (2005).

[4] M. Tumminello, C. Coronnello, F. Lillo, S. Miccichè, R.N. Mantegna, Spanning trees and bootstrap reliability estimation in correlation –based networks, International Journal of Bifurcation and Chaos 17, 2319 – 2329 (2007).

[5] M. Tumminello, F. Lillo, R.N. Mantegna, Kullback-Leibler distance as a measure of the information filtered from multivariate data, Phys. Rev. E 76 (3): Art. No. 031123 (2007).