CONNECTIONS BETWEEN ENTROPY AND THEORY OF FINANCE

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Central financial quantitative properties very frequently exhibit fat tails, e.g., in probability distributions, in time-dependent functions, among others. Where do they come from? It seems plausible that many of them emerge from diversified scale-invariant aspects of the relevant phenomena. The nonadditive entropy S_q, and its associated nonextensive statistical mechanics, appear to be natural generalizations of the Boltzmann-Gibbs-von Neumann-Shannon (additive) entropy S_{BG}, and its associated statistical mechanics. This generalized entropy, which recovers S_{BG} for $q = 1$, is intimately connected with (multi)fractals. It is therefore no surprise that many relevant financial quantities (return distributions, volume distributions, relaxation functions, among others) exhibit the functional forms that emerge within the nonextensive theory, such as *q*-exponentials, *q*-Gaussians, and their extensions. The mesoscopic mechanisms that are involved include multiplicative noise, inhomogeneous and/or nonlinear diffusion coefficients, non-Markovian processes, and related processes. All of them are, at a more microscopic scale, related with breakdown of ergodicity, i.e., discrepancy between ensemble averages and time averages. A brief introduction to these connections, as well as those concerning the recent *q*-generalization of the Central Limit Theorem, will be presented. Several current applications will also be mentioned.

References

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