Risk theory with Non-Poissonian arrivals

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Abstract

A classical problem in Risk theory is studying the evolution of an insurance company surplus $U_t = U_0 + vt - S_t$ whenever premiums are received at a certain constant rate v and it is assumed that the number and amount N_t and S_t of corporation losses resulting from claims occurring in $[0, t]$ can be modeled by a Poisson and compound Poisson process, respectively.

Here we consider a natural generalization of this classical setting to the case where S_t is a continuous time randon walk (CTRW), i.e., a pure jump process, non necessarily Poissonian. CTRWs generalize in an important way the latter processes allowing for general distribution of the waiting times. Correlation between waiting times and jumps is also permitted.

In Statistical physics CTRWs became popular after the work of Montroll & Weiss [1], and have been used to describe physical phenomena ever since. To list a few examples we note applications to earthquake modelling (e.g., [2, 3]) and rainfall description [4]. More recently, the use of CTRWs has been advocated to give a microscopic, tick-by-tick, description of financial markets: see [5, 6, 7, 8].

Note that, when arrivals are non-Poissonian, the Markov property is no longer preserved; as a consequence most basic statistics depend on whether or not the present is one of the jump times (see [9]).

We are interested in the mean time for the surplus U_t to reach a fixed value b or else get bankrupt *assuming that the only available information is the present state* $U_r = x, x < b$. The mean exit time is found to be given in terms of two objects. The first of these is the "excess life" of classical renewal theory (see [1, 10]), whose distribution is given in terms of Laplace transforms. For Erlang distributed waiting times the inversion is performed in an explicit way. The ideas are extended to cover also the case of the Gamma distribution.

The second object alluded to above is the mean time $\mathbb{T}(x)$ for the surplus U_t to exit $(0, b)$ right after a a claim has arrived. We use a probabilistic reasoning to derive an integral equation that $\mathbb{T}(x)$ must satisfy. For a certain class of waiting times we show how the equation *can be solved in closed form.* We note that partial results when waiting times have Erlangian distribution have already been derived, under the assumption that *the present r is one of the jumping times*(see [11]; see also [9] for related ideas).

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