Priority queues with scale-free arrivals of incoming tasks

N. Masuda[a,b], **J. S. Kim**[c], and **B. Kahng**[c]

[a]Graduate School of Information Science and Technology, The University of Tokyo
7-3-1 Hongo, Bunkyo, Tokyo 113-8656, Japan
masuda@mist.i.u-tokyo.ac.jp

[b]PRESTO, Japan Science and Technology Agency
4-1-8 Honcho, Kawaguchi, Saitama 332-0012, Japan

[c]FPRD, Department of Physics and Astronomy, Seoul National University
Seoul 151-747, Korea
jk@phya.snu.ac.kr
bkahng@snu.ac.kr

The interevent time distribution of human activities in many situations follows a power law with exponent 1 or 3/2. This fact can be explained by a queueing model operating under priority-based protocols [1,2]. In the priority queue model, each incoming task is assigned a priority value, and the task with the highest priority is executed first. The distribution of the interevent time corresponds to the waiting-time distribution of tasks in the queue [1,2]. The power-law behavior with exponent 1 or 3/2 emerges when the queue length is finite or infinite, respectively. Previous studies focused on the case in which incoming tasks are delivered to the queue independently of each other. Then, the number of incoming tasks per unit time follows the Poisson distribution. However, the number of emails received by a single user per unit time sometimes follows a power-law distribution [3]. In this presentation, we report the analytical solution of the queue-length distribution and the waiting-time distribution for this case [3], using the generation function formalism (e.g. [4]).
Our discrete-time model is described as follows. In each time step, the task with the highest priority in a queue is executed with probability μ. Simultaneously, n tasks arrive to the queue, where n is distributed according to a power law $\lambda_n \propto \lambda n^{-\gamma}$ $(n > 0)$, $\lambda_0 = 1 - \lambda$, where $0 < \lambda < 1$. Each task is assigned a priority x uniformly distributed on [0,1] without loss of generality. The queue length is unbounded. This model generalizes the model by Grinstein and Linsker [5,6], which corresponds to $\lambda_0 = 1 - \lambda$, $\lambda_1 = \lambda$, and $\lambda_n = 0$ for $n \geq 2$ in our model.
Our results are summarized as follows. Given $\mu > \langle n \rangle_\lambda (1 - x)$, where $\langle n \rangle_\lambda$ is the average number of incoming tasks per unit time, the steady state for the queue-length distribution exists for $\gamma > 2$. Then we obtain

$$\tilde{Q}_x(m) \sim \frac{1}{m^{\gamma-1}} \quad (m \to \infty), \tag{1}$$

where $\tilde{Q}_x(m)$ is the probability that the queue has m tasks whose priorities are larger than x in the steady state. Particularly, the steady state exists regardless of the value of x when $\mu > \langle n \rangle_\lambda$. The mean queue length denoted by $\langle m(x) \rangle_{\tilde{Q}}$, which exists only when $\gamma > 3$ (see Eq. (1)), is equal to

$$\langle m(x)\rangle_{\tilde{Q}} = \frac{2(1-\mu)\langle n\rangle_\lambda(1-x) + (\langle n^2\rangle_\lambda - \langle n\rangle_\lambda)(1-x)^2}{2(\mu - \langle n\rangle_\lambda(1-x))}. \quad (2)$$

The interevent time distribution follows a power law $P_{\rm w}(\tau) \sim \tau^{-\alpha}$ with α given as follows. When $\langle n\rangle_\lambda < \mu$, $\alpha = \gamma - 1$ for $\gamma > 2$. When $\langle n\rangle_\lambda = \mu$, $\alpha = (2\gamma - 3)/(\gamma - 1)$ for $2 < \gamma \leq 3$, and $\alpha = 3/2$ for $\gamma > 3$. When $\langle n\rangle_\lambda > \mu$, the results for $\langle n\rangle_\lambda = \mu$ apply, but only the tasks with priority $x > (\langle n\rangle_\lambda - \mu)/\langle n\rangle_\lambda$ are executed.

Keywords
priority queue, power-law, human behavior, social dynamics

References

[1] A.-L. Barabási, "The origin of bursts and heavy tails in human dynamics," *Nature*, v. 435, p. 207–211, 2005.
[2] A. Vázquez *et al.*, "Modeling bursts and heavy tails in human dynamics," *Phys. Rev. E*, v. 73, 036127, 2006.
[3] N. Masuda, J. S. Kim, and B. Kahng, "The waiting-time distribution in priority queues: heterogeneous number of incoming tasks," Preprint: arXiv:0805.0841
[4] N. Masuda, K. -I. Goh, and B. Kahng, "Extremal dynamics on complex networks: analytic solutions," *Phys. Rev. E*, v. 72, 066106, 2005.
[5] G. Grinstein and R. Linsker, "Biased diffusion and universality in model queues," *Phys. Rev. Lett.*, v. 97, 130201, 2006.
[6] G. Grinstein and R. Linsker, "Power-law and exponential tails in a stochastic priority-based model queue," *Phys. Rev. E*, v. 77, 012101, 2008.