MODELING TSALLIS DISTRIBUTIONS BY NONLINEAR STOCHASTIC DIFFERENTIAL EQUATIONS WITH APPLICATION TO FINANCIAL MARKETS

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Studies of financial markets using statistical physics approach revealed some stylized facts, including the power-law tails of probability densities of the trading activity and log-returns as well as long-term power-law correlations of the traded volume, absolute value returns and returns standard deviation (the volatility). The financial observables may be related to the Tsallis' statistical approach (see, e.g., [1-3] and references herein), resulting in the power-law tails, as well. On the other hand, trades in financial markets occur at discrete times and may be considered as point processes [4-6]. Here, starting from the multiplicative point process model [7] and nonlinear stochastic models [8, 9] of $1/f$ noise and power-law distributions we derive nonlinear stochastic differential equations

$$\mathrm{d}x = \Gamma(x_m + x)^{2\eta-1}\mathrm{d}t + (x_m + x)^{\eta}\mathrm{d}W. \tag{1}$$

and

$$\mathrm{d}x = \Gamma(x_m^2 + x^2)^{\eta-1}x\mathrm{d}t + (x_m^2 + x^2)^{\eta/2}\mathrm{d}W. \tag{2}$$

generating processes with the q-exponential and q-Gaussian distributions of the observables with the long-range power-law autocorrelations and $1/f^{\beta}$ power spectral density. Here W is is a regular Wiener process and Γ, x_m, and $\eta > 1$ are the parameters.
Further we analyze analytically and numerically the properties of solutions of these equations in relation with the nonextensive statistical mechanics framework and relevance of the generalized and adapted equations for modeling of the financial processes.

Keywords
stochastic models, power-law, point processes, Tsallis statistics, financial markets

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