STATISTICAL STEADY STATES OF RANDOM TRANSPORT ON RANDOM SCALE-FREE NETWORKS

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For a given network structure we consider random transport of a conserved quantity. We assume that a portion of the conserved quantity defined on each node moves to a neighbor node which is chosen randomly among those nodes connected directly by links. Repeating this type of finite portion random transport we can generally observe a non-trivial statistically steady state if the portion, denoted by θ, takes a value between 0 and 1. Two of the authors (M.T. and H.T.) studied the basic properties of this random transport phenomenon in both cases of regular lattices and the mean-field in which all nodes are directly connected [1]. It is known that in the limit of θ goes to 0 the quantity distributes uniformly on all nodes with infinitesimal Gaussian fluctuations. In the limit of θ goes to 1 large fluctuation remains in the statistically steady state characterized by a power law distribution that is similar to the steady power laws observed in aggregation with injection systems. In the intermediate case $0<\theta<1$ there exists a non-trivial statistically steady state with a fat-tailed distribution between Gaussian and a power law.

In this paper we first numerically made random networks characterized by power law distribution of link numbers, and we observed the steady state distribution by numerical simulation. For such scale-free networks the steady state distribution is not uniform even in the vicinity of $\theta=0$ but it follows a power law characterized by the link number distribution as known by the theory of Page-rank [2]. In the limit of θ goes to 1 we confirmed that the steady fluctuation converges to a universal power law that is equivalent to the mean-field case, and in the intermediate case we always observe a strongly fat-tailed distribution as typically shown in Fig.1.

As an application in economy we apply this transport model to a real network of companies in Japan consisted of about 1 million nodes. We show that the steady distribution for different values of θ characterizes some properties of the network structure.

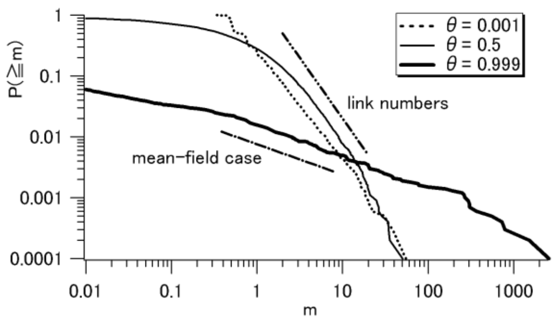

Fig.1 Cumulative distribution of the steady state fluctuation of transported quantity, m, on a scale-free network consisted of 10,000 nodes. The steady distribution depends on the portion θ. Guided lines show distribution of link numbers and the power law for the mean-field case of aggregation system with injection.

Keywords
complex network, scale-free network, random transport, power law distribution

References

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