Time scale defined by fluctuations and multifractal time in foreign exchange markets

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In this contribution, a new time scale is defined by price fluctuations observed at a given resolution. Mandelbrot defined a time-scale, called trading time which - compared to clock time - flows slowly during some period and fast during others (see [4,5]). Let t be clock time and $T(t)$ a trading time, typically multifractal time. Fractional Brownian motions in multifractal time is represented as $X(t) = B_H(T(t))$ where B_H denotes fractal Brownian motions.

Given a time series g(t), local maxima determined by a resolution scale C along price axis are defined as follows [1]. If there exist $d_1, d_2 > 0$ such that $g(s-d_1) < g(s)-C$, $g(s+d_2) < g(s)-C$, and g attains its maximum at s in $[s-d_1, s+d_2]$, then $g(t)$ has a C-local maximum at s. The scale C is the accuracy of measuring the extreme values. The ordinary local maximum is the case $C = 0$: 0-local maximum. Likewise, if there exist $d_1, d_2 > 0$ such that $g(s-d_1) > g(s)-C$, $g(s+d_2) > g(s)-C$, and $g(s) = \min_{s-d_1 < t < s+d_2} g(t)$, then $g(t)$ has a C-local minimum at s. Given a resolution C, the time series $g(t)$ attains its C-extreme values (C-local maximum or C-local minimum) at $s_1^C, s_2^C, s_3^C, \cdots$. Let R(C) denotes the sum of the absolute variation between neighboring C-extreme values: $R(C) = \sum |g(s_{i+1}^C) - g(s_i^C)|$. The fold dimension which is a kind of latent dimension defined from the relation: $R(C)/C \sim C^{-D_f}$ is independent of time scale (see [2,3]). Given resolution C, a time scale: "C-fluctuation" time can be defined as $\tau_C(t) = \sharp(\{s_i^C | s_i^C < t\})$, where $\sharp()$ denotes counting measure. The time intervals between these neighboring C-extreme values can be measured by real-time (clock time) and tick-time (the order of submission).

In this contribution, intraday fractal structures and the relations of the three time scales in foreign exchange markets are analyzed. The data set used is the bid and ask prices of foreign exchange rates; US dollar (USD)/Japanese yen (JPY), USD/Euro (EUR), and EUR/JPY. The accuracy of the data is one minute and quotes within a minute are recorded in order of submission. The intervals between these C-extreme values are exponentially distributed when they are measured by tick-time.

Keywords
exchange rates, time scale, fractal, multifractal time, high-frequency data

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