Scale Dependence of Increment Probability Density Function in S&P500 Index

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Over the past decade, applications of statistical physics methods to economics have attracted considerable interest in the physical and economics science communities. In particular, the suggested similarities between the market crash and a phase transition, and market crashes as critical point phenomena, have held the promise of understanding the dynamics of market crashes and the possibility better to evaluate risk in the market [1].
To date, the volatility of stock price changes has been used as a measure of how much the market is liable to fluctuate, which is of interest to traders because it quantifies the risk, and it is the key input in the option pricing model by Black and Scholes.For this reason, the statistical properties of volatility have been intensely studied by economists, and recently by physicists [2]. However, it is impossible to explain the occurrence of extremely large fluctuations by estimating only volatility. As we demonstrate in this contribution, for risk analysis it is necessary to quantify not only volatility, but also the non-Gaussian nature of price fluctuations on a relatively short timescale ($\sim$10 min).
In addition, we observe an abrupt transition in the scale dependence of the non-Gaussian PDF when the Black Monday crash occurs, which supports a recently proposed interpretation that the Black Monday crash was triggered by a critical phenomenon. The scale-invariant properties in the non-Gaussian PDF have been considered as a characteristic feature observed at the critical point.
To date, in order to demonstrate the similarities between a market crash and a phase transition, deterministic and long timescale (day-to-day) variation patterns, such as log-periodic oscillations, of stock indices have been studied [1]. However, these results seem unreliable, because the fitting procedure mainly depends on subjective judgment, and determinism is unlikely to be expected in the financial market.
In our contribution, as illustrated in Fig 1, using the recently introduced methodology of analysis of scale dependence of increment probability density [3], following Kiyono et al [4], we provide comprehensive evidence of the occurrence of a phase transition and critical behavior in the dynamics of a financial index.
Further, by analyzing the dynamical temporal evolution of the critical regime, we demonstrate the empirical fact that a precursor of the October 1987 crash was observed in the fluctuations on a relatively short timescale $\sim$10 minutes.
We believe that our findings solidify the picture of market crashes as a critical phenomenon and give insights which will stimulate further research into the physics of market regulation.

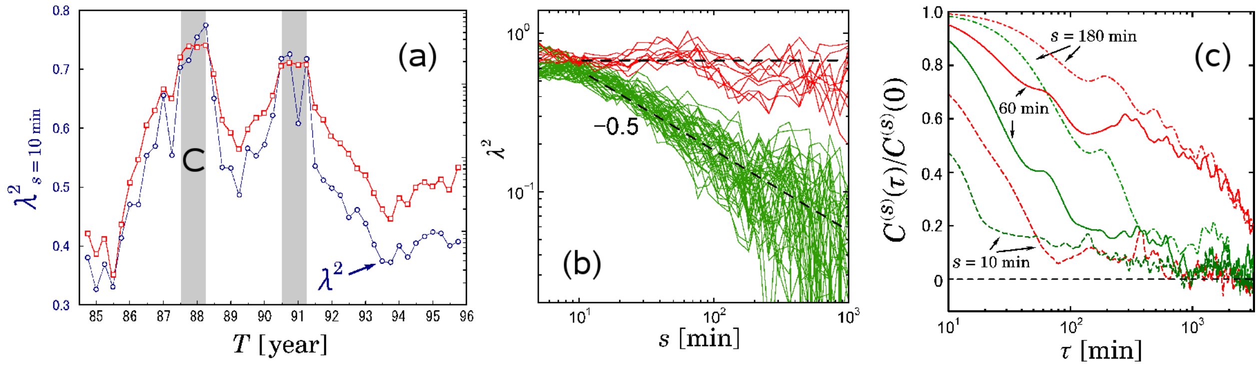

Figure 1: (a) The temporal dependence of the Castaing's model based non-Gaussianity index λ^2 [3,4,5] and volatility σ over a one-year time span ($\sim 5 \times 10^4$ data points at sampling intervals at $\Delta t = 2$ min) of index evolution [3,4]. The left, gray region contains Black Monday in October 1987. (b) The scale dependence of λ^2. Red lines correspond to the results in the critical region C, characterized by scale invariant non-Gaussian PDF. The data before Black Monday (disjoint from region C) is represented by green lines [4]. (c) Magnitude correlation functions [4,6], $C^{(s)}(\tau)/C^{(s)}(0)$ corresponding to (green), and (red) in (center figure), for $s = 10, 60, 180$ min.

Keywords
multiscale time series fluctuation analysis, risk assessment, phase transition, criticality.

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