Approaches to the financial market crisis from the viewpoint of PUCK model

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Recently, one of the authors (M.T.) and her group introduced a new type of random walk model called potentials of unbalanced complex kinetics (PUCK). In this model, market price, {P(t)}, is described by a random walker in a temporally deforming potential force whose center is given by the trace of the walker[1].

$$P(t+\Delta t) - P(t) = -\frac{d}{dp}U(p,t)|_{p=P(t)-P_M(t)} + f(t) \tag{1}$$

where, $P_M(t) = \frac{1}{M}\sum_{j=0}^{M-1} P(t-j)$ and $U(p,t) = \sum_{k=1}^{\infty} \frac{b_k(t;M)}{k} p^k$. In ordinary market state, average of price fluctuation is nearly equal to zero and potential coefficients other than $b_2(t)$ are negligibly small. In this paper, we show two approaches of detecting market crisis in its early stage based on this PUCK model.

We already showed the appearance of higher-order potential force, i.e, $U(p,t) = \frac{b_1(t;M)}{2}p^2 + \frac{b_\gamma(t;M)}{\gamma+1}p^{\gamma+1}$ and its statistical significance in financial market data at crisis[2]. Theoretically the existence of such higher order potential terms indicates the possibility of occurrence of monotonic directional motion of market prices, which will lead a bubble or a crash.

In the continuum limit, $\Delta t \to 0$, of Eq.(1), the PUCK model is described by the following Langevin equation with time-dependent mass and viscosity.

$$-M\frac{b_2(t;M)}{6}\frac{d^2P(t)}{dt^2} + \frac{1}{\Delta t}\left(1 + \frac{b_2(t;M)}{2}\right)\frac{dP(t)}{dt} = -\frac{b_1(t;M)}{\Delta t^2} + \frac{f(t)}{\Delta t^2} \tag{2}$$

In usual physical situations mass and viscosity never takes a negative value, however, we show that viscosity can take a negative value when prices move exponentially during bubbles and crashes. There is a possibility that we can detect the start of market crisis from the value of this market viscosity. We can also show that market prices reproduced by Eq. (3) satisfies the various stylized facts of financial market, such as power law price change distribution, very short decay of autocorrelation of price changes, long correlation of volatility, and abnormal diffusion of price.

Keywords Financial crisis, Higher-order potential force, Langevin process, Negative viscosity

References

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