Network Motifs in Inter-firm Network

T. Ohnishi[a], **H. Takayasu**[b], and **M. Takayasu**[c]

[a] Graduate School of Law and Politics, The University of Tokyo
7-3-1 Hongo, Bunkyo-Ku, Tokyo 113-0033, Japan
ohnishi@sat.t.u-tokyo.ac.jp

[b] Sony Computer Science Laboratories
3-14-13 Higashigotanda, Shinagawa-ku, Tokyo 141-0022, Japan

[c] Department of Computational Intelligence and Systems Science,
Interdisciplinary Graduate School of Science and Engineering, Tokyo Institute of Technology
4259-G3-52 Nagatsuta-cho, Midori-ku, Yokohama 226-8502, Japan

To understand economic activity such as dynamics of firm's growth, system-level study of complex interactions is important. Essence of the system lies in dynamics and it cannot be described by merely investigating components of the system. Thus, beyond the characteristics of individual component, empirical analysis of the network structure of interactions is necessary. Nevertheless, it is only recently that these analyses have been performed by using huge amount of electronically stored data in economic system [1, 2].

Many systems of interacting elements arising in physical, sociological, and biological areas can often be abstracted to complex networks. Traditional measures of topology of complex network, such as clustering coefficient, the shortest path, and others, have been widely employed. These measures are intended for undirected network or ignore link directions. However, to understand the economic system deeper, information about the direction of interactions should be also taken into account [3].

Subgraphs that occur more frequently than expected in random are referred to as motifs, while those occurring less frequently are anti-motifs. Recently, network motifs have attracted attention as a tool to study directed networks [4]. In biological networks, a small set of network motifs appears to serve as elementary building blocks of transcription networks from bacteria to mammals. Such motifs can be regarded as functional modules. Different networks usually display different motifs, and motifs can be used to characterize families of networks [5].

We analyze Japanese inter-firm network which consists of about one million nodes (firms) and four million directed links (supplier-customer relations). To detect 3-node motifs, we first generate randomized networks, each of which has the same single node characteristics as the real network: Each node in the randomized networks has the same number of inlinks, outlinks and bidirectional links as the corresponding node has in the real network. These randomized networks preserve the same number of appearances of all 2-node subgraphs as in the real network. This ensures that detected significant deviation is independent of significant subpatterns. To generate randomized networks, we apply the Markov-chain Monte Carlo switching algorithm to the real network on which randomly chosen pairs of connections are repeatedly switched (e.g., $X1 \to Y1$, $X2 \to Y2$ are replaced by $X1 \to Y2$, $X2 \to Y1$) until the network is well randomized [6]. Then we count all combinations of 3-node subgraphs of real and randomized networks and determine the appearances of all possible motifs [7]. The statistical significance is evaluated by estimating the empirical p-value derived from the empirical distribution of the count.

In the analyzed inter-firm network, clique , where all firms are reciprocally interconnected, shows the strongest motif. This result is very similar to results in social networks and World Wide Web hyperlink networks. Furthermore, V-shaped triads , which are common motifs in word-adjacency networks and bipartite model networks, are also network motifs. This is related to the structure of inter-firm relationships in which a firm tends to transacts other firms in a different business category. On the other hand, feedback loop  is an anti-motif, implying that feedback is not important in production network. These empirical findings provide valuable insight into the relation between the economic function and the local structure of the inter-firm network. More detailed investigations are in progress to understand the production processes that yield given motifs.

Keywords
inter-firm network, complex network, network motif, randomized network, topological role

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