THE EXPONENTIAL ORNSTEIN-UHLENBECK MODEL: ANALYTICAL AND NUMERICAL RESULTS

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We analyze the problem of the analytical characterization of the probability distribution of financial returns in the exponential Ornstein-Uhlenbeck model with stochastic volatility [1] [2] [3]. In this model the prices are driven by a Geometric Brownian motion, whose diffusion coefficient is expressed through an exponential function of an hidden variable Y governed by a mean-reverting process. We derive closed-form expressions for the probability distribution and its characteristic function in two limit cases. In the first one the fluctuations of Y are larger than the volatility normal level, while the second one corresponds to the assumption of a small stationary value for the variance of Y.

Theoretical results are tested numerically by intensive use of Monte Carlo simulations. The effectiveness of the analytical predictions is checked via a careful analysis of the parameters involved in the numerical implementation of the Euler-Maruyama scheme and is tested on a data set of financial indexes. In particular, we discuss results for the German DAX30 and Dow Jones Euro Stoxx 50, finding a good agreement between the empirical data and the theoretical description [4].

Keywords

Ornstein-Uhlenbeck processes, Fokker-Planck equations, stochastic volatility models, financial returns, Monte Carlo methods

References

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