SYMBOLIC SHADOWING AND THE COMPUTATION OF ENTROPY FOR OBSERVED TIME SERIES

Diana A. Mendes[a] , **Vivaldo M. Mendes**[b], **Nuno Ferreira**[c] and **Rui Menezes**[c]

[a]Corresponding Author: Department of Quantitative Methods, ISCTE
and UNIDE, Avenida Forças Armadas, 1649-026 Lisbon, Portugal
diana.mendes@iscte.pt

[b]Department of Economics, ISCTE and UNIDE
Avenida Forças Armadas, 1649-026 Lisbon, Portugal
vivaldo.mendes@iscte.pt

[c]Department of Quantitative Methods, ISCTE and UNIDE
Avenida Forças Armadas, 1649-026 Lisbon, Portugal
nuno.ferreira@iscte.pt, rui.menezes@iscte.pt

Order, disorder and recurrence are common features observed in complex sequences that can be observed in many fields, like finance, economics, biology, physiology and other natural phenomena. On the other hand, these can be produced by chaotic dynamical systems and one way to analyze them is via symbolic dynamics, a mathematical-statistical technique that allow the detection of the underlying topological and metrical structures in the time series.

Symbolic dynamics [3] is a powerful tool initially developed for the investigation of discrete dynamical systems. The main idea consists in constructing a partition, that is, a finite collection of disjoint subsets whose union is the state space. By identifying each subset with a distinct symbol, we obtain sequences of symbols that correspond to each trajectory of the original system.

One of the major problem in defining a “good” symbolic description of the corresponding time series, is to obtain a generating partition, that is, the assignment of symbolic sequences to trajectories is unique, up to a set of measure zero. Unfortunately, this is not a trivial task, and, moreover, for observed time series the notion of a generating partition is no longer well defined in the presence of noise.

In this paper we apply symbolic shadowing [2], a deterministic algorithm using tessellations, in order to estimate a generating partition for two distinct time series and consequently to compute their metric and topological entropies. This algorithm allows producing partitions such that the symbolic sequences uniquely encode all periodic points up to some order.

The first time series we consider is generated by an iterated function system (IFS). We choose this kind of map because generating partitions are quite well understood for IFS and there are rigorous mathematical methods to compute metric and topological invariants ([4],[5]), like entropies, escape rates and fractal dimensions. The methods used in this case are based on weighted Markov matrices. Hence, it is easy to compare the results obtained by symbolic shadowing and Markov matrices.

The second time series we consider is a financial time series, the Portuguese Stock Index (PSI20).

We also apply the symbolic shadowing algorithm in order to compute entropies and we compare these results with those obtained by considering the Pesin's identity (the metric entropy is equal to the sum of positive Lyapunov exponents). To obtain the Lyapunov exponents, we reconstruct the state space of PSI20 by applying an embedding process [A] and also use the Wolf *et al.* [6] algorithm. Moreover, we compute the Rényi entropy, since the symbolic block entropies are special cases of Rényi entropies.

Keywords
symbolic dynamics, shadowing, time series analysis, nonlinearity

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