Dynamical Analysis of Cross Correlations and Systemic Risk in the Japanese Stock Market

Tetsuya TAKAISHI[1], Zeyu ZHENG[2] and H.Eugene STANLEY[3]

[1] Hiroshima University of Economics, 5-37-1, Gion, Asaminami-ku, Hiroshima, 731-0192 Japan
[2] Shenyang Institute of Automation, Chinese Academy of Sciences,Shenyang 110016, P.R.China
[3] Center for Polymer Studies and Department of Physics,Boston University, Boston, MA 02215, USA
[1] E-mail: tt-taka@hue.ac.jp

Abstract

We study cross correlations constructed from the return time series of 366 stocks traded on the Tokyo Stock Exchange from January 5, 1998 to December 30, 2013. The correlation matrices are calculated with a rolling window of 400 days. The dynamical behavior of correlation matrices shows that the average of off-diagonal elements abruptly increases at the volatile market stages. We detect three volatile market stages corresponding to the bankruptcy of Lehman Brothers, Tohoku Region Pacific Coast Earthquake and FRB QE3 reduction observation. By applying the principal component analysis we show that the cumulative risk fraction also increases abruptly at the same volatile market stages. Thus the rise of systemic risk can be captured by a rapid increase in matrix elements and the cumulative risk fraction.

Keyword: Correlation matrix, Principal component analysis, Cumulative risk fraction, Systemic risk

1 Introduction

The stock market is a complex system that undergoes unstable periods causing financial crisis in some cases. Measuring systemic risk is an important task to monitor current market status and possibly to avoid future financial crisis. Several approaches have been used on measuring systemic risk such as the principal component analysis (PCA)[1, 2, 3, 4] and the Granger-causality test[1, 3] In this study we employ the principal component analysis (PCA) for cross correlations between stocks on the Tokyo Stock Exchange and analyze the dynamical properties of the cross correlations.

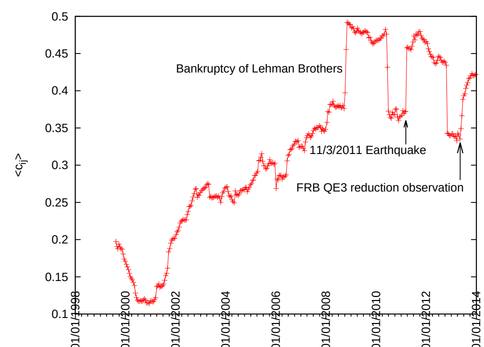

Figure1: Average off-diagonal elements of the cross correlation matrix. Each average is taken over 400 days in the rolling window.

2 Cross correlation matrix

In order to study the dynamical properties of correlation matrices we analyze the daily price data of stocks traded on the Tokyo Stock Exchange from January 5, 1998 to December 30, 2013, that corresponds to 5905 working days. We selected 366 stocks from the list of the Topix 500 index under the criterion that they have at least 4000 working-day data.

Let $r_i(t)$ be a return for stock i at time t defined by

$$r_i(t) = \ln p_i(t) - \ln p_i(t-1), \tag{1}$$

where $p_i(t)$ is the price for stock i at t. Using return time series $r_i(t)$ the equal-time cross correlation matrix is calculated as

$$c_{ij} = \frac{\langle r_i(t) - \langle r_i(t)\rangle\rangle\langle r_j(t) - \langle r_j(t)\rangle\rangle}{\sigma_i \sigma_j} \tag{2}$$

where σ_i is the standard deviation of $r_i(t)$ and $\langle\rangle$ stands for an average taken over a period of the rolling window. In this study we take a rolling window of 400 working days that roughly corresponds to two years.

Fig.1 shows the dynamical evolution of the average off-diagonal matrix element $\langle c\rangle$ given by

$$\langle c\rangle = \frac{2}{N(N-1)}\sum_{i>j}^{N} c_{ij}. \tag{3}$$

where $N = 366$. From the figure we recognize

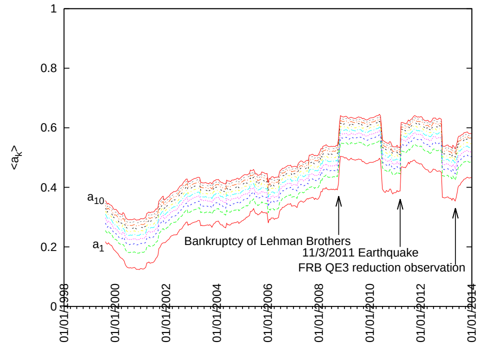

Figure2: Time evolution of the CRF with 400-day rolling window.

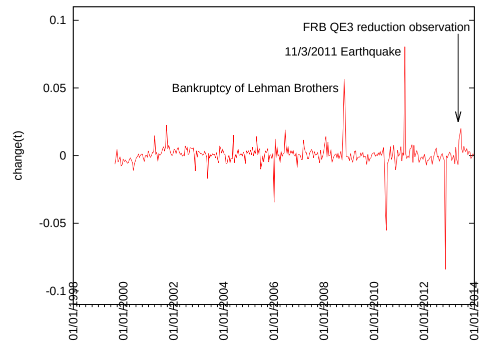

Figure3: Change of the CRF. Here time evolution of $change_1(t)$ is presented.

that there exist three points where $\langle c \rangle$ increases abruptly. According to the historically observed events these points correspond to the bankruptcy of Lehman Brothers, 11/3/2011 Tohoku region pacific coast earthquake and the FRB QE3 reduction observation as indicated in the figure. It has been also observed that the Japanese stock market enters the volatile stage at these points.

3 Dynamical behavior of eigenvalues

In order to further investigate the dynamical properties of the cross correlation matrices we apply the PCA. The PCA has been also used to measure the systemic risk[1, 2, 3, 4]. First we compute the eigenvalues of the cross correlation matrices, denoted as $\lambda_1, \lambda_2, \ldots, \lambda_N$ where all eigenvalues are sorted as $\lambda_1 > \lambda_2 > \ldots > \lambda_N$. Then we calculate the cumulative risk fraction (CRF) defined by[1, 2]

$$a_k = \frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{N} \lambda_i}. \tag{4}$$

This quantifies the system variance explained by the first k principal components over the total variance[2].

Fig.2 shows the evolution of the CRF for $k = 1, \ldots, 10$. We find that the CRF also increases abruptly at the same points observed in the average matrix elements.

We can also quantify the points by the change of the CRF defined by[3]

$$change_k(t) = a_k(t+1) - a_k(t). \tag{5}$$

The time evolution of $change_1(t)$ is presented in Fig.3. It is found that the change of the CRF shows pronounced positive spikes at the same points. Note that large negative spikes are artificially caused by the period of the rolling window.

4 Conclusion

We have analyzed the cross correlation matrices of 366 stocks traded on the Tokyo Stock Exchange from January 5, 1998 to December 30, 2013. We find that both the average off-diagonal elements of the cross correlation matrices and CRF show abrupt increases at three points that correspond to three volatile stages of the Japanese stock market: the bankruptcy of Lehman Brothers, Tohoku Region Pacific Coast Earthquake and FRB QE3 reduction observation. The change of the CRF also identifies these three points. The cross correlation matrices contain relevant information on the systemic risk of financial markets. By carefully analyzing the dynamical properties of the cross correlations we could monitor the risk of financial markets.

References

[1] M. Billio, M. Getmansky, A.W. Lo and L. Pelizzon, "Econometric measures of connectedness and systemic risk in the finance and insurance sectors", J. Financial Econ. 104, 535 (2012).

[2] M. Kritzman, Y.Z. Li, S. Page and R. Rigobon, "Principal Components as a Measure of Systemic Risk", J. Portf. Manag. 37, 112 (2011).

[3] Z. Zheng, B. Podobnik, L. Feng and L. Pelizzon, "Changes in Cross-Correlations as an Indicator for Systemic Risk", Sci. Rep. 2, 888 (2012).

[4] F. Ren and W.Z. Zhou, "Dynamic evolution of cross-correlations in the Chinese stock market", arXiv:1309.1154.