Intra-day variability of the stock market activity and stationarity of the financial time series

Mateusz Wilinski[1] and Tomasz Gubiec[2]

[1,2] Faculty of Physics, University of Warsaw, Warsaw, Poland.

E-mail: [1]mateusz.wilinski@fuw.edu.pl, [2]tomasz.gubiec@fuw.edu.pl

Abstract

We describe the impact of the intra-day activity pattern on the autocorrelation function estimator. We obtained a strict formula connecting estimator of the autocorrelation functions of non-stationary process and its stationary counterpart. By mean of this formula, we proved that day seasonality of intertransaction times extends the memory of both process itself and it's absolute value extending the relaxation time.

Keyword: autocorrelation, intraday pattern, stationarity

Introduction

Time series of logarythmic price returns are commonly used in broad range of financial analysis. Intraday type of this data have lately gained particular interest. While using many different types of estimators and models, it is often assumed, directly or indirectly, that underlying processes are stationary. Unfortunately it is not the case for financial time series, even for logarythmic returns. There are at least few well known reasons against financial data stationarity. One of them is *volatility clustering*, a positive autocorrelation observed for different measures of volatility.

Another aspect, more closely associated to our work, are different types of seasonalities, that can be seen in various time scales. A major example is so called 'lunch' effect which refers to day trading and is characterized by high volatility and short intertrade times right at the beginning of the session and just before closing of the quotations, and significantly lower volatility in the middle of the day. Intra-day changes of the activity on stock market is a well-known empirical fact observed all around the world on different types of market. More details can be found in Hasbruck [1], Chan *et al.* [2], Admati and Pfleiderer [3].

Previously, researchers focused on the variability and seasonality of volatility only. Some of them tried to measure its impact and remove the effect from data [4]. That doesn't affect time intervals between trades at all, whereas we observe that this effect is more significant. The aim of this work, is to find how the day seasonality, observed in transactions time intervals, affects the time series autocorrelation. To do so, we will propose a time transformation, that will eliminate this seasonality from analysed process, and try to analytically describe the 'lunch' effect.

Our main result is a strict dependence between autocorrelation function estimators of stationary process and its seasonal (non-stationary) transformation. Furthermmore, we found that for financial time series, adding seasonality of time intervals extends the memory of underlying process but at the same time, it doesn't create any additional autocorrelations.

References

[1] Hasbrouck, J., *Empirical market microstructure: The institutions, economics and econometrics of securities trading.* New York: Oxford University Press, 2007, **4**.

[2] Chan, K. C., Christie, W. G., and Schultz, P. H., *Market structure and the intraday pattern of bid-ask spreads for NASDAQ securities.* Journal of Business, 1995, 35–60.

[3] Admati, A., and Pfleiderer, P., *A theory of intraday patterns: Volume and price variability.* Review of Financial Studies, 1988, **1**, 3–40.

[4] Amaral, L. A., Cizeau, P., Gopikrishnan, P., Liu, Y., Meyer, M., Peng, C. K., and Stanley, H. E., *Econophysics: can statistical physics contribute to the science of economics?* Computer physics communications, 1999, **121**, 145–152.