The Fluctuations of Stock Market in a Quantum Paradigm

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Abstract

To study the fluctuations of stock market in a quantum paradigm instead of classical statistical formalism, a quantum-like model is proposed to simulate the dynamics of a single stock price in the stock market. The price return is considered a virtual particle in quantum paradigm and can be described by the vectors (wave functions) in the Hilbert space. The wave function, whose square modulus is interpreted as the probability density, moves complying with the Schrodinger equation (SE). Taking collective influence of traders' behaviors into account, we construct the Hamiltonian with a financially meaningful potential for the price return. We compare the quantum-like model with classical ones and show that the former is effective in stock market.

Keyword: fluctuations, stock market, quantum paradigm, price return, Schrodinger equation (SE)

1. Introduction

The mathematical formalism of classical physics has been widely applied into a wide range of fields. However, be similar to that the classical physics inevitably became unqualified for the microscopic systems, Newtonian formalism is also inadequate for the explanation of other domains. So the scientists introduced quantum theory into the relevant disciplines[1] and even social science[2,3]. Our work is the application of quantum theory into finance, specifically the stock price in a stock market.

2. Model

In a stock market, the price return x of a stock can be considered a virtual quantum particle that moves under SE. If the Hamiltonian of the particle is time-independent, the SE can be reduced to

$$H(x)\psi(x) = E\psi(x),$$

$\psi(x)$ is the wave function of the particle, $|\psi(x)|^2$ represents the probability of finding the particle at x.

Using the financial explanation of the potential by Bouchaud and Cont[4], we introduce a Hamiltonian to describe a price return in a liquid market with the form of:

$$H(x) = -\frac{1}{2m}\frac{d^2}{dx^2} + \left(ax^2 + bx^3\right),$$

where we use natural unit. a, b are constants reflect the influence of the trader's behaviors, and m is the virtual mass for the price return.

We solve the time-independent Schrodinger equation numerically with the potential morphing method (PMM) proposed by M. Rieth[5].

3. Results

The ground state wave function is used to reproduce the Gaussian-like probability distribution of the price return, which also can be achieved by classical models. Besides the ground state, the wave functions of higher energy levels make it possible to explain some exotic time series of the stock price.

Compared to the classical models, it is obvious that the quantum-like model has simpler and more understandable form. With the help of the numerical algorithm, we can construct different potentials including more market information to study the corresponding stock market.

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