## Non-Parametric Simulated ML Estimation of the Heterogeneous Agent Models SMSEC2014 Template (for an oral presentation)

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## Abstract

We adapt and test a new method for empirical validation of heterogeneous agent models (HAMs) and estimate the very famous Brock and Hommes (1998) [1] financial HAM. We suppose the methodology is likely to appear more general for other HAMs or ABMs in the future. The methodology of the Non-Parametric Simulated Maximum Likelihood Estimation (NPSMLE) is based on the work of Kristensen and Shin (2012) [2]. For the HAM the analytical form of the density function does not exist and thus we cannot use the MLE method. However, we are still able to simulate observations from that model to non-parametrically estimate the conditional density by kernel methods. The likelihood function is then constructed and we obtain NPSMLE via maximization. We test our methodology via Monte Carlo simulations and present the small finite sample properties results both of the single parameter estimation of the intensity of choice and the simultaneous multiple parameter estimation.

**Keyword:** Behavioral Finance, Heterogeneous Agent Model, Heterogeneous Expectations, Intensity of Choice, Non-Parametric Maximum Likelihood Estimation, NPSMLE.

In recent financial literature, the Efficient Market Hypothesis, Representative Agent Approach and the Rational Expectation Hypothesis, dominating in the past, are often being replaced by more realistic agent based computational approaches. This methodology aims to highlight that although consequences of market fluctuations are worldwide, the essence of problems remains on the level of individual market agents with their heterogeneous beliefs and expectations.

Core idea of the heterogeneous agent models (HAMs) is the well documented and systematic human departure from agents' full rationality towards bounded, limited rationality. HAMs therefore employ interacting groups of boundedly rational agents to model the financial world. In these systems the prices are driven endogenously based on interaction of agents and so far this reasonably realistic methodology appears very successful in replicating observed stylized facts of financial data.

Proponents of the HAM methodology often suggest the courageous medium-term attempt to complement or even alternate the mainstream DSGE approach for the policy making. For this to happen, it is inevitable to be able to estimate these models on empirical data to use them for forecasting. Generally, there are two crucial complications or rather challenges in estimating the HAMs. The first is the highly nonlinear nature and complexity of these systems, prohibiting researchers of using classical estimation methods. The second is the high number of degrees of freedom and parameters which need to be set or estimated, together with a number of different possible settings requiring huge computational capacity for the analysis. On one hand the empirical validation is an important part of any modeling cycle, but on the other not many attempts exist so far on estimation of HAMs.

The aim of this paper is to make a methodological step further and propose a method together with a computational algorithm for empirical validation of a particular HAM design. We present results of the estimation of the most famous and widely analyzed financial HAM of Brock and Hommes (1998) [1] but we suppose the methodology is likely to appear more general and useful for other HAMs or generally agent-based models (ABMs) e.g. in Macro in the future.

Our methodology of the Non-Parametric Simulated Maximum Likelihood Estimation (NPSMLE) of the Brock and Hommes (1998) [1] model is based on the work of Kristensen and Shin (2012) [2]. For the HAM the analytical form of the density function does not exist and therefore we are not able to analytically derive the likelihood function to maximize the log-likelihood and estimate the model parameters.

However, we are still able to simulate observations from that model. These observations are used to non-parametrically estimate the conditional density by kernel methods. The likelihood function is then constructed and we obtain NPSMLE via maximization over the parameter space.

Important theoretical properties of the estimator for dynamic models: the consistency and asymptotic efficiency have been proved under rather general assumptions Kristensen and Shin (2012) [2]. We check the validity of these assumptions also for the HAMs.

We test our methodology via Monte Carlo simulations and analyze the effect of various bandwidths for the kernel estimation of the conditional density. We present the small finite sample properties results of the single parameter estimation of the intensity of choice (beta parameter of switching between trading strategies) and show that the consistency and asymptotic efficiency of the estimator is reached. We also present the results of the multiple parameter estimation in which we simultaneously estimate the intensity of choice and standard deviations of normal distributions from which the agent's beliefs are randomly generated.

We also broaden our work to the sensitivity analysis of results to various extensions of the model. We analyze the effect of agents' memory, number of trading strategies, absence of the fundamental strategy, or the impact of the intensity of the stochastic noise in the system.

The ultimate goal of our effort is the estimation of the model using real data from world stock markets and subsequent comparison to other 'competing' approaches to assess whether HAMs can outperform in terms of fitting the real financial data and the possible forecasting performance power. This is, however, left for the future work

## References

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