Co-action and Rational Cooperation: A novel solution framework for Prisoner's Dilemma and other symmetric games

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Abstract

The conventional approach to solve non-cooperative games is through the concept of Nash equilibrium, a strategy choice by each player where no player can do better by deviating unilaterally from it. Here we explore an alternate framework for solving such games having a symmetric payoff structure that we refer to as *Co-action equilibrium.* The equilibrium is unique for all two-person, payoff-symmetric games and may be more efficient than the standard Nash equilibrium in certain cases. We analyze three well-known two-person games, viz., Prisoner's Dilemma (PD), Hawk-Dove and Stag Hunt using the concept. We show that for PD, in co-action equilibrium, agents cooperate when the temptation (the payoff to the defecting player when the other cooperates) is low, while for larger temptation, they use a probabilistic strategy. For the Hawk-Dove game, the agents play 'Dove' (non-aggressive) when the possible benefit on playing 'Hawk' (aggressive) is low, while switching to a probabilistic strategy when the reward for being aggressive is high. For the Stag Hunt game, we show that agents always opt for cooperation and achieve the most efficient outcome under this solution concept. We extend the co-action solution to multi-player iterative games by solving a specific example of an N-player PD. We show the existence of non-trivial steady states where a majority of cooperating agents coexist with a minority of defecting agents for a range of values for temptation and reward for mutual cooperation.

Keyword: Symmetric games, Nash equilibrium, Co-action equilibrium, Prisoner's Dilemma

Games represent strategic interaction between agents where the term agents could refer to humans, animals or inanimate objects like computers. Concepts and ideas from game theory have been applied to a wide variety of fields ranging from economics and political science to computer science and evolutionary biology where we study the behavioral decisions of individuals and their collective outcome. In its simplest two-person form, a game constitutes a set of actions called strategies available to each of the players and a payoff for each possible outcome. Fig. 1 shows the payoff matrix corresponding to a general two-person game with each agent having two possible strategies (A and B) that they can choose from. A rational agent is assumed to choose that strategy which will maximize her payoff. In order to arrive at such a strategy, one uses a solution concept, which is a formal rule or a meta strategy for predicting how a game will be played between rational agents. It is assumed that each agent knows that their opponents are also rational and would be taking this into account in their decision about strategy choice (referred to as the assumption of common knowledge of rationality). The solution for a game is the strategy that agents will employ under this assumption given the payoff structure of the game. For non-cooperative games on which we focus, the conventional solution concept used is that of Nash equilibrium. Informally, a Nash equilibrium is a strategy choice by each player where each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy [1].

In this work, we present an alternate solution concept namely the co-action equilibrium, which was introduced in the limited context of a minority game in Ref. [2]. Here we expand the concept to analyse all symmetric games (here symmetric means payoff-symmetric, see Fig. 1). It presents an alternate view point about the assumption of common knowledge of rationality and has advantages over the standard Nash equilibrium in that it is unique and often gives more socially efficient outcome without abandoning the individual payoff-optimizing nature of the agents.

In the co-action concept, each agent in a 2-person payoff-symmetric game realizes that both

		Player 2	
		Action A	Action B
Player 1	Action A	(a,b)	(c,d)
	Action B	(e,f)	(g,h)

Figure1: A generic payoff matrix for a two-person game where each agent has two strategies A and B available to her. For a payoff-symmetric game $a = b$, $g = h$, $c = f$ and $d = e$.

of them are in a symmetric situation. Thus, whatever choice she is going to make using howsoever a complex process, her opponent being in the same symmetric situation and being equally rational, will make exactly the same choice. This concept of rational behavior is similar to an earlier, non-formal definition of rationality provided by Hofstadter in the context of Prisoner's Dilemma (PD) [3], where it was argued that rational agents playing PD will always cooperate. However, unlike the philosophical argument put forward by Hofstadter, the co-action concept described here provides a formal and general framework for solving symmetric games under this interpretation of rationality. In fact, under co-action equilibrium, the solution to PD is not to cooperate always (as suggested by Hofstadter) but to cooperate with a probability when the temptation to defect is large.

We analyse three well-studied two-person symmetric games, namely, PD, Hawk-Dove (also known as Snowdrift or Chicken) and Stag hunt. We illustrate in detail the differences in prediction between the Nash solution and the co-action solution for these games. In PD, the agents will fully cooperate for low temptation payoffs whereas they randomize between cooperation and defection for higher temptation payoffs. Thus for higher temptation payoffs, the co-action concept gives a probabilistic answer in contrast to the Nash solution (which always leads to defection) and Hofstadter's intuition (which suggests that agents will cooperate always). It is interesting to note that in various experiments of PD with human players, the observed level of cooperation is non-zero but non-perfect (see for e.g Ref. [4]). To the best of our knowledge, there is no solution concept which yields a strategy involving probabilistic cooperation for the one-shot PD game. Similarly, for Hawk-Dove game, we show that agents adopt risk-averse behavior when the stakes are low (i.e., reward for aggressive behavior is low) whereas they use a probabilitistic strategy - randomly switching between hawk (aggressive) and dove (non-aggressive) behavior with a certain probability - when the stakes are high. For Stag hunt, we show that agents always opt for mutually co-operating behavior, which is also Pareto optimal, under the concept of co-action equilibrium.

We also discuss how the concept can be applied to iterated, multiplayer symmetric games by considering the specific example of iterated Prisoner's Dilemma (IPD). We show that rational agents following the co-action equilibrium solution concept will adopt a win-stay lose-shift strategy [5] in two-player IPD. In the N-player IPD, where each agent plays with everybody else in a pair-wise interaction in each round, we show that non-trivial steady states occur where a group of cooperators who form the majority coexist with a group of defectors depending upon the ratio of the temptation to the reward for cooperation.

References

[1] M. J. Osborne and A. Rubenstein, *A course in game theory*, MIT Press, Cambridge, MA (1994).

[2] V. Sasidevan and D. Dhar, Physica A, **402** 306 (2014).

[3] D. Hofstadter, Scientific American **248** 6 (1983). Reprinted in: Hofstadter, Douglas (1985), Metamagical Themas. Basic Books. pp. 737755.

[4] J. Andreoni and J. Miller, The Economic Journal **103** 570 (1993).

[5] M. Nowak and K. Sigmund, Nature **364** 56 (1993).