# Social inequalities in probabilistic labor markets 

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#### Abstract

We discuss social inequalities in labor markets for university graduates in Japan by using the Gini and $k$ indices. Feature vectors which specify the abilities of candidates (students) are built-into the probabilistic labor market model [1]. Here we systematically examine what kind of selection processes (strategies) by companies according to the weighted feature vector of each candidate could induce what type of large inequalities in the number of informal acceptances leading to the large mismatch between students and companies.


Keyword: Labor markets, Social inequality, Probabilistic model, Agent-based simulations, Diversity

Inequalities are unavoidable phenomena in any competitive society and it sometimes gives us a strong motivation to work harder. However, at the same time, inequality might cause some sort of 'instabilities' in our socioeconomic systems. For instance, such social inequalities might emerge even in simultaneous recruiting of new graduates in Japan, in particular, in the number of informal acceptances resulting in quite large the 'mismatch'. In our previous modeling of labor markets [1], we simply assumed that each company selects the students 'randomly' up to their quota. However, it is obviously far from reality.

In this paper, we attempt to modify the model by taking into account realistic selection procedures by companies. Here we assume that each company $k(=1,2, \cdots, K)$ possesses their own valuation basis, say, a superposition of $M$ orthogonal bases $\boldsymbol{e}_{1} \equiv(1,0 \cdots, 0), \cdots,(0, \cdots, 0,1) \equiv \boldsymbol{e}_{M}$ as $\boldsymbol{Y}^{(k)}=y_{1}^{(k)} \boldsymbol{e}_{1}+\cdots+y_{M}^{(k)} \boldsymbol{e}_{M}=\left(y_{1}^{(k)}, \cdots, y_{M}^{(k)}\right)$. On the other hand, each student $i(=1,2, \cdots, N)$ possesses their own 'feature vector' which is also given by a superposition of bases as $\boldsymbol{X}^{(i)}=x_{1}^{(i)} \boldsymbol{e}_{1}+\cdots+x_{M}^{(i)} \boldsymbol{e}_{M}=\left(x_{1}^{(i)}, \cdots, y_{M}^{(i)}\right)$. Therefore, the total score of student $i$ made by the company $k$ is now given by a projection to the basis $\boldsymbol{Y}^{(k)}$ as

$$
\begin{equation*}
s^{(i, k)}=\boldsymbol{X}^{(i)} \cdot \boldsymbol{Y}^{(k)} \tag{1}
\end{equation*}
$$

If the student $i$ is scored as the top $v_{k}^{*}$-ranking, where $v_{k}^{*}$ stands for the quota of the company $k$, among all candidates who submitted their application letters to the company $k$, the student $i$ might obtain the informal acceptance. In this sense, the selection procedure is 'unified' when all companies choose the 'weights' as $y_{1}^{(k)}=1, y_{2}^{(k)}=$ $\cdots=y_{M}^{(k)}=0$. In such a specific case, a few top scored students in terms of the basis $x_{1}^{(i)}$ might 'monopolize' the informal acceptances, and as the result, the mismatch between students and companies are considerably enhanced.

Recently, a lot of companies are trying to get talented persons from various aspects to accelerate the 'diversity' in their companies. Hence, a unified selection basis does not catch the reality and here we assume that $\left(y_{l}^{(k)}, x_{l}^{(i)}\right), l=$ $1, \cdots, M$ are identically independent distributed variables in terms of a normal $y_{l}^{(k)} \sim N(0, \sigma)$ and a lognormal $x_{l}^{(i)} \sim$ $\left(1 / \sqrt{2 \pi \tau^{2}} x_{l}^{(i)}\right) \mathrm{e}^{-\left(\log x_{l}^{(i)}-\mu\right)^{2} / 2 \tau^{2}}$ for $N, K \gg 1$.

For these artificial settings of 'diversities', we carry out agent-based simulations using our probabilistic labor markets [1] and evaluate the social inequalities by means of
inequality measures, the Gini and $k$ indices [2]. The $k$ index which was recently introduced denotes the situation in which $k$ percentage of students shares totally $(1-k)$ percentage of the informal acceptances in the labor market. The Gini index $g$, which is given by an area surrounded by what we call Lorentz curve: $(X(r), Y(r)), X(r) \equiv$ $\int_{0}^{r} P(m) d m, Y(r) \equiv \int_{0}^{r} m P(m) d m / \int_{0}^{\infty} m P(m) d m$ for the distribution of number of acceptance $P(m)$ and the perfect equality line $Y=X$, is quantified by the $k$ index as

$$
\begin{equation*}
2 k-1 \leq g \leq 2 k(2-k)-1-(1-k)^{2}\left(\xi(k)+\xi(k)^{-1}\right) \tag{2}
\end{equation*}
$$

with $\xi(k) \equiv X^{-1}(k) / Y_{0}, Y(0) \equiv \int_{0}^{\infty} m P(m) d m$. Namely, the Gini index $g$ is bounded from the both above and below by means of the $k$ index (see also [2] for the details).

We can improve the bounds so as to make them much titer ones recursively as $g_{\text {lower }}(n), g_{\text {upper }}(n), n=0,1,2, \cdots$, where $g_{\text {lower }}(0), g_{\text {upper }}(0)$ correspond to the bounds in (2). Several preliminary results are shown in Figure 1. From this figure, we clearly find that $g_{\text {lower }}(n), g_{\text {upper }}(n)$ converge to the exact value as the step $n$ increases. The details of the analysis would be reported at the conference.


Figure1: Gini index $g$ as a function of the degree of ranking preference $\gamma$ of students in the model [1]. The inset shows $g_{\text {lower }}(n), g_{\text {upper }}(n), n=0,1,2$. Here we set $N=1000, K=$ $50, a=5, \alpha=1.62, \beta=1$ for the definitions in the model [1].

## References

[1] H. Chen and J. Inoue, Evolutionary and Institutional Economics Review 10, No. 1, pp. 55-80 (2013).
[2] J. Inoue, A. Ghosh, A. Chatterjee and B. K. Chakrabarti, arXiv:1406.2874 (preprint).

