Jump-diffusion model of bubbles and crashes with non-local behavioral self-referencing

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Abstract

Keyword:

Most existing jump-diffusion processes of financial bubbles and crashes, in particular the class of rational-expectation bubble models, implement the no-arbitrage condition to obtain that the instantaneous conditional risk premium (or return) is proportional to the contemporaneous crash hazard rate. Within bubble-crash models, this embodies and extends the standard risk-return relationship. We argue that the condition matching instantaneously return and risk is unrealistic and misleading because it assumes perfect markets and no friction, which are conditions likely to be even less true in times of exuberant bubbles and of punishing crashes. We propose a novel class of jump-diffusion models in which the hazard rate of correcting jumps is determined by a non-linear S-shaped function of a non-local estimation of mispricing. Specifically, the mispricing is estimated as an exponential moving average of the difference between the present and the past log-prices over a long time scale, typically one year or more. This specification is rooted in behavioral finance, exploiting in particular the traits of "anchoring" on past price levels and on "probability judgement" about the likelihood of a correction as a function of the amplitude of the self-referential mispricing. The insights obtained from analytical and numerical simulations of the model are remarkable. In addition to the standard stylised facts, rising markets are understood as transient regimes when the risk of negative jumps is undersampled while the investors expect a sufficiently large remumeration to compensate for the large risk they anticipate. This makes quantitative the adage that "markets climb a wall of worry". Reciprocally, our model cures the fundamental problem of crash jump models, which are in general rejected by data because they assume that crashes occur in a single large negative jump, by describing correctly that correction regimes and crashes are also phases with a significant duration, with inter-dependence between the sequences of corrections mediated by the interplay between the price and jump hazard rate dynamics. We derive analytically the likelihood function and develop expectation-maximisation estimation methods that are implemented on synthetic time series and real markets. As a significant bonus, we show that the model calibration provides a robust estimation of the risk premium, event in bearish markets, which is generally hidden. Our model provides a novel understanding of the risk-return relationship resulting from the entanglement of diffusion and jump risks.