# Study on Transport Costs across a Network Using a Minority Game Model with Imperfect Information 

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#### Abstract

We study how the average cost in a route choice problem changes when the perfect information assumption is relaxed to an imperfect information one. As a result of the simulations, we found a peculiar range in which the average transport cost under the imperfect information assumption becomes lower than that under the previous assumption of perfect information.


Keyword: Agent-Based Model, Minority Game, Transport, Congestion, Networks, Imperfect Information

## 1. Introduction

Minority Game (MG)[1][2] is often used in the study of complex adaptive systems. It has also been applied to route choice problems on transport networks[3].

Using MG, Gourley et al. studied the problem on a simple network with a hub (Fig.1), in particular, on how the averaged transport cost changes with different congestion charges[3]. In their model, all agents can get the congestion history of the hub at every time step. Hence the model can be defined as a Perfect Information Model (PIM). However, in the real world, agents who had no experience of using the hub route may not be able to know the hub history. In this study, we introduce an Imperfect Information Model (IIM) and will investigate the influence of this informational imperfectness on the average transport cost through the simulations.

## 2. Model

### 2.1. The Previous Model

Here, the model in the previous study [3] will be described briefly. The transport network used in the study is shown in Figs.1(a) and 1(b). Each agent will move from a node to another node on the circle. If an agent can use the central hub, she shall decide whether to use the central hub or not. If she can not use the central hub, she would be forced to use the peripheral pathway.


Fig. 1 The network used in the model. (a) An agent who can use the central hub choose whether to take the hub pathway or peripheral pathway. (b) An agent who can not use the central hub is forced to take the peripheral pathway.

Transport costs are calculated by the following equations, where $C_{\text {central }}$ is the cost of the hub pathway, and $C_{\text {outside }}$ is that of the peripheral pathway.

$$
\begin{align*}
C_{\text {central }} & = \begin{cases}1 & \text { if } N_{\text {central }} \leq L \\
1+c c & \text { if } N_{\text {central }}>L\end{cases}  \tag{1}\\
C_{\text {outside }} & =n \tag{2}
\end{align*}
$$

Symbol $n$ is the number of links that the agent has to pass by, $c c$ the congestion charge, $N_{\text {central }}$ the number of agents who use the central hub and $L$ the hub capacity.

As an MG agent, she has a memory length of $m=2$ and a set of information-based strategies of $s=2$. Here the information is referred to the history of congestion in the hub that an agent can remember. For PIM, all agents
could get the congestion history of the hub regardless of whether they had used the hub or not. The information is represented by a series of m-bit Boolean strings, namely $00,01,10$ and 11 , where bit 1 stands for the congestion state and bit 0 stands for no congestion state. Each of the strategies is evaluated by the payoff, that is, the transport cost. If a strategy could give a lower cost, it will receive an increment of score. If a strategy gave a higher cost, it would receive a decrement of score. Agents always use the strategy with the highest score.

### 2.2. The Imperfect Information Model

In IIM, we assume that only those agents who have used the hub can get the history at the time step. For agents who have not used the hub, they have to replace the Boolean bit with a "?" in the history. Meanwhile, the scores of strategies own by those ignorant agents will not be updated.

For agents who have the history string including "?", their action will be determined by drawing randomly from the actions corresponding to possible history strings in the higher score strategy. For example, given a history string 1 ?, the possible history strings would be 11 and 10 , hence the action would be selected randomly out of the two actions corresponding to these history strings.

## 3. Results and Discussion

Results of a typical set of simulations are shown in Fig.2, whose horizontal axis is for the number of agents who can access the hub ( $\theta$ ), and the vertical for the difference between average cost per agent in PIM and that in IIM (d).


Fig.2. Each simulation is run for 10,000 time steps with each value of $\theta$ and we did 1,000 simulations for each value of $c c$.

The cost difference is negative from $\theta=0$ to $\theta=60$,
but after that the difference becomes positive for a certain range. The size of this peculiar range depends on the congestion cost of the hub. The reason why such a positive difference exists will be unraveled in the study. Lastly, when most of the agents can access the hub route, the cost difference becomes negative again.

## 4. Summary

We extend the previous MG model for the route choice problem on a network by considering the imperfectness of information. In the simulations, we have found three ranges for the cost difference of PIM and IIM. It is interesting that PIM is not always advantageous in the view of average transport cost.

## References

[1] W. Brian Arthur, "Inductive Reasoning and Bounded Rationality", American Economic Review, Vol. 84, No. 2, May, (1994) 406-411
[2] D. Challet, Y.-C. Zhang, "Emergence of cooperation and organization in an evolutionary game", Physica A 246 (1997) 407-418.
[3] Sean Gourley, Neil. F. Johnson, "Effects of decision-making on the transport costs across complex networks", Physica A 363 (2006) 82-88.

