Do connections make systems robust?
–A new scenario for the complexity-stability relation–

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Abstract

Whether interactions and/or connections make system robust or fragile has been a central issue in broad range of field. Here I show a novel type of mechanism which governs the robustness of open and dynamical systems, using a very simple mathematical model. This mechanism suggest a moderate number (~ 10) of interactions per element is optimal to make the system robust against unpredictable attack.

Keyword: Robustness, Fragility, Stability, Complex Systems, Open/Evolving Systems, Economy, Community, Bottom-up scheme

Most real complex systems of our interest, such as social, economic, engineering, and biological systems, are *ecosystem-like*: in those systems, constituting elements are not fixed and the complexity emerges (or at least persist) under successive introductions of new elements. Those systems sometimes grow in its complexity and/or size, but also sometimes collapse. How and when in general, such open systems can evolve toward complex structure under the successive inclusions remains an open question.

Here we tackle this classical problem using the following extremely simple model. It is found that systems under this process either evolve toward infinitely large system or stay finite, depending on the unique parameter m: the average number of interactions per element. Interestingly, this transition originates from the balance of the two effects. Although having more interactions makes each element robust, it also increases the impact of the loss of an element.

This novel relation might be a origin of the fact that we often find moderately sparse (order of 10 average degree, not 1) networks in real complex systems. The present minimal model also yields non-trivial and realistic distribution function of the lifetime of elements.

(The Model)

0. (Create an initial state.)

1. Calculate the fitness for each species: $f_i = \sum_{j}^{incoming} a_{ij}$

2. If the fitness of the species are all positive, go to the next step. If not, delete the species of minimum (and therefore negative) fitness and then re-evaluate the stability of the system:
   (i) Delete the species
   (ii) Delete the links connecting to and from that species.
   (iii) Re-evaluate the extinction: go back to the step 1.

3. A new species is added to the system. After that, go back to the first step to simulate the intrinsic dynamics on the new community.
   (i) The new species has m new links.
   (ii) The interacting species are chosen randomly from the resident species, with equal probability $1/N(t)$.
   (iii) The direction of the link is also determined randomly with equal probability 0.5 for each direction.
   (iv) The weights of the links are drawn from the standard normal distribution.

References

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