Study of Multi-Asset Stock Markets with an Agent-Based Model

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Abstract

In this study, we utilize an agent-based model for the construction of artificial multi-asset stock markets which can offer insights on the mechanism of price formation and the origin of cross-correlations between assets. As the first part of our work, we conduct an empirical analysis using random matrix theory on cross-correlations of the daily return of 1245 Japan stocks for 10 years period. As the second part of our work, we construct an artificial multi-asset stock market with an auction-type market clearing mechanism in reference to Genoa Artificial Stock Market[2)]. As a preliminary result, the main important stylized-facts of univariate and multivariate price processes are reproduced. In addition, we confirm that the largest eigenvalue deviated from random matrix theory can be interpreted as the "market response" and investigate some factors contributing to its deviations.

Keyword: Random Matrix Theory, Stylized-facts, Agent-based model, Econophysics.

1. Introduction

Since the advent of econophysics, a number of financial stylized-facts (SFs) have been discovered through the application of statistical physics approaches to the quantitative analyses of financial data. Nevertheless, most research focus on univariate SFs which disregards the cross-correlations between financial assets. Hence, crucial ingredients of modern finance *e.g.* risk diversification via portfolio optimization, which require a precise estimation of the cross-correlation matrix $\boldsymbol{C}$, are often overlooked in this field.

With this background, we aim to study the nature and origin of SFs of multi-asset market relevant to $\boldsymbol{C}$ through agent-based models (ABMs).

2. Multivariate stylized-facts

2.1. Data Analyzed

L=2299 daily returns of N=1245 Japan stocks, which are listed in 1st Section of Tokyo Stock Exchange, with a period of 10 year (2002-2012) are extracted from Yahoo!Finance and the cross-correlation matrix of normalized returns of stocks $\boldsymbol{C}=(c_{ij})$ is calculated and analyzed in RMT's framework.

2.2. Results of Empirical Analysis

Empirical analysis shows that, in terms of spectrum of eigenvalues (SE) and distribution of eigenvectors (DE), most eigenvalues (90%) and its corresponding eigenvector of $\boldsymbol{C}$ agree with RMT's prediction. However, some eigenvalues deviate significantly (2% from the upper limit and 8% for the lower limit from RMT's prediction bound). Particularly, the value of the largest eigenvalue λ_{max} of $\boldsymbol{C}$ is 440, which is 130 times larger than RMT's prediction. Most of our results are in good consistence with the empirical analysis of US stock market.[1)] Hence, the properties of SE and DE of $\boldsymbol{C}$ can be considered as a universal SF and may be used as a benchmark for the construction of an artificial multi-asset stock markets (AMASM). Meanwhile, an explanation of the nature of such deviation is also needed.

3. Artificial Multi-Asset Stock Markets

3.1. Introduction of the Model

An AMASM model in reference to the Genoa Artificial Stock Market (GASM)[2)] is constructed. In this ABM, agents are free to trade J companies' stocks and issue transaction orders associated with a limit price for each financial asset based on their own strategy yet limited by their finite wealth. All the orders issued are collected by a central auction-type market-maker, the clearing price and transaction volume will be determined at the intersection of supply and demand curves.

As the simplest yet non-trivial case, we consider an AMASM consisting of random-agents assigned with heterogeneous memory (time window T_i) for observing the past market's performance. Agents will randomly allocate a portion of their current financial resources to all the stocks with a random weight.

In order to validate the model, we define a Market-Index (MI) $P^{\mathrm{index}}(\mathrm{h})$ as

$$P^{Index}(h) = \frac{\sum_{j=1}^{J} P^j(h)\, V^j}{\sum_{j=1}^{J} V^j},$$

where $P^j(h), V^j$ are denoted respectively as the price of stock j at the time-step h, the total amount of stocks j available in the whole market.

3.2. Simulation Conditions

All the 65 random-agents, who are endowed with equal initial financial resources ($i.e.$ 20^{10} unit of cash, 1000 unit of stocks for 200 different companies' stocks with the same initial value worth of 10^5 unit of cash), and random T_i s ranging from 10 to 120, trade in the AMASM for 2000 time steps. In addition, we investigate two cases in which agents are influenced by the MI or not. If agent i ignores the MI, the allocation of wealth on stock market is denoted by $W_{\mathrm{i}}^{\mathrm{risky}}$, and it is determined by $W_{\mathrm{i}}^{\mathrm{risky}}(h+1) = \gamma\, W_i(h)$, where $\gamma \sim \mathrm{U}(0,1)$. Otherwise, W_i^{risky} is determined by

$$W_i^{\mathrm{risky}}(\mathrm{h}+1) = A_i(h) W_i(h),$$

$$A_i(h) = \begin{cases} \gamma + 0.2, & I_i(h) > 0 \\ \gamma - 0.2, & I_i(h) < 0 \end{cases} \quad \text{s.t} \quad A_i(h) \in [0,1],$$

and $I_i(h)$ denoted as the average return of MI for the period of past time step T_{i}.

3.3. Simulation Results and Discussion

With regards to the univariate SFs, this model can be utilized to reproduce important SFs *e.g.* volatility clustering *etc.*, in consistence with previous studies[2].

With respect to multivariate SFs, specifically SE, one eigenvalue (λ_{max}) was found to deviate from the RMT's prediction indicating that stocks are not completely independent of each other despite the fact that all agents act randomly. Upon closer inspection, the answer to that deviation of λ_{max} lies in the finiteness of financial resources.

λ_{max} is often interpreted as the "market response"[1]. We confirm this intepretation through ABM with the following two arguments. First, when a portfolio is built based on the components of eigenvector corresponding to the λ_{max}, the portfolio's return exhibits highly similar behavior to the return of the P^{Index}. Second, when the multi-asset market is considered as a one-factor model, it can be derived analytically that the cross-correlation coefficient between all different stocks c_{ij} can be expressed as $c_{ij} = (\lambda_{max} - 1)/(J\text{-}1)$ for $\forall i \neq j$. We confirmed that c_{ij} can be estimated as such through ABM.

Lastly, we consider the factors that influence λ_{max}. When agents ignore (consider) the MI, the λ_{max} is almost 20 times (33 times) larger than RMT's prediction. Behaviour of agent's taking the market response, *i.e.* MI into consideration could be a factor contributing to the deviation of λ_{max}.

4. Conclusions

In this work, we confirm that the λ_{max} can be intepreted as the "collective market response" and the deviation can be affected when agents are influenced by the market's feedback, *i.e.* the MI through ABM.

References

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